CS738: Advanced Compiler Optimizations

SSAPRE: SSA based Partial Redundancy Elimination

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- ▶ Remove totally redundant computations (CSE)

PRE without SSA

- Iterative data flow analysis

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- Operates on control flow graph

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- Computes global and local versions of data flow information

SSAPRE

- Information flow along SSA edges

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- ▶ Information flow along SSA edges
- ▶ No distinction between global and local information

SSAPRE: Challenge

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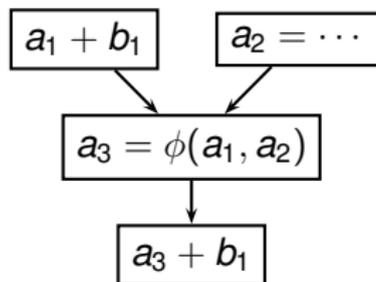
- ▶ Here $a_1 + b_1$ is same as $a_3 + b_1$ when control follows the left branch. Lexically different, but computationally identical

SSAPRE: Key Idea

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- ▶ PRE on SSA form of RCVs (*h*) to remove redundancies
- ▶ Final program will be in SSA form

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  - ▶ Ignoring the version number of the operands
  - ▶ In the earlier example, $a_3 + b_1$ and $a_1 + b_1$ could be identical

SSAPRE Steps

- Six step algorithm

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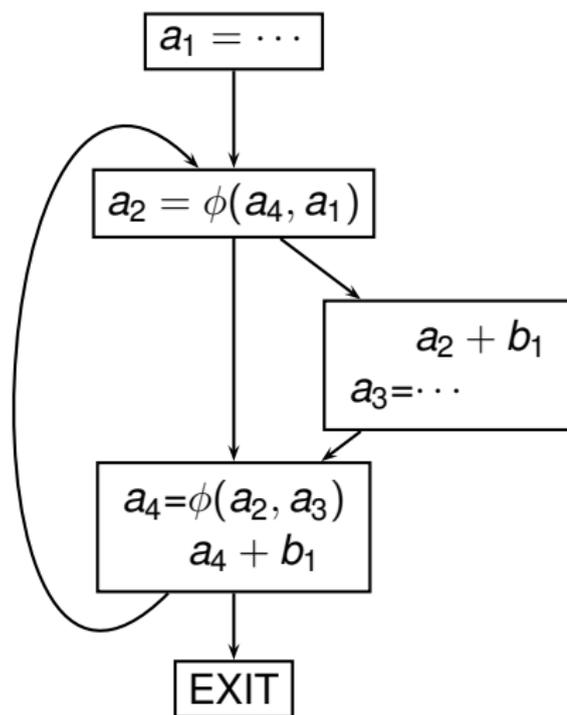   4. WillBeAvail computation

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  6. Code Motion

Running Example

Φ-insertion

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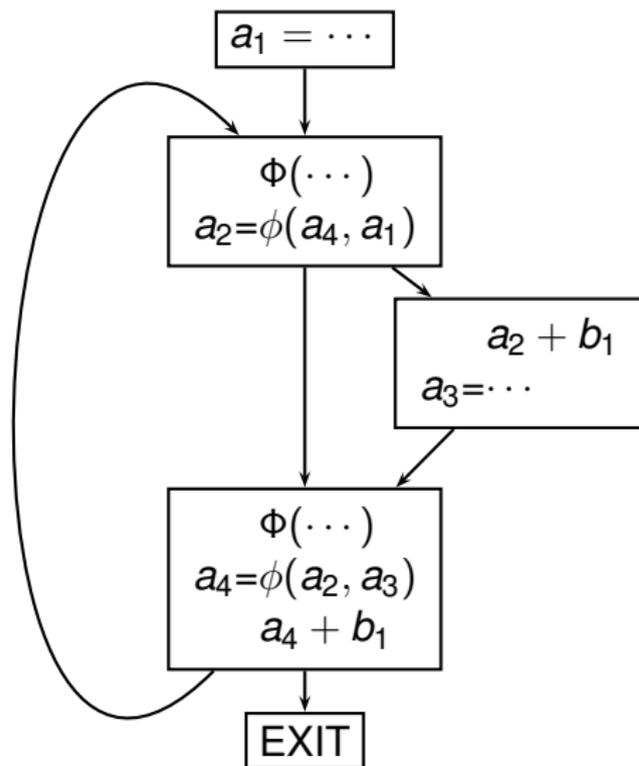- At iterated dominance frontiers of occurrences of E
- At each block having a ϕ for some argument of E
  - Potential change in the expression's value

Φ-insertion

Rename

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- ► After renaming
    - ► Identical SSA instances of *h* represent identical values of *E*
    - ► A control flow path with two different instances of *h* has to cross either an assignment to an operand of *E* or a Φ of *h*

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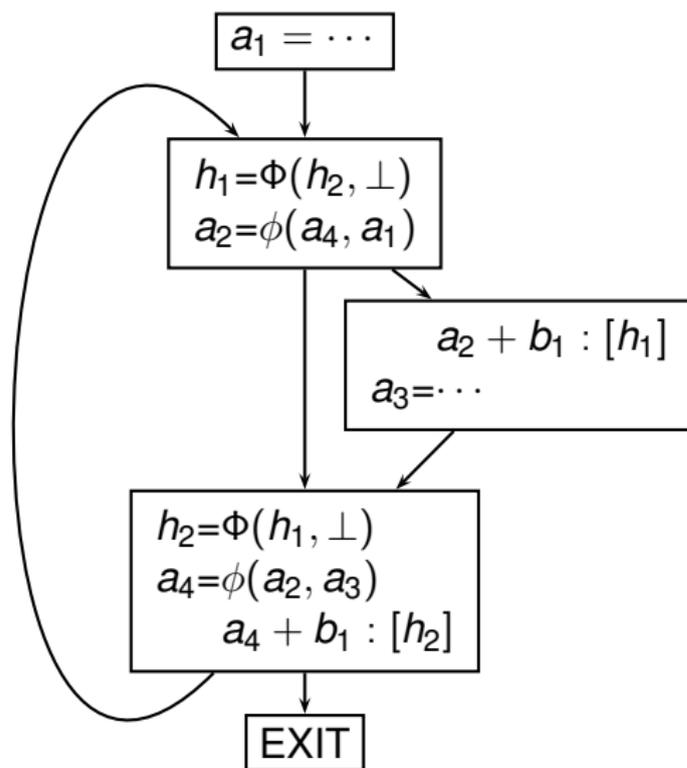   - ▶ If all match, *h* gets same version as the top of *E* stack

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  - ▶ if E is operand of Φ, in the corresponding predecessor block
    - ▶ for each operand of E, compare the version of operand with the top of the rename stack for operand
    - ▶ If all match, h gets same version as the top of E stack
    - ▶ If any mismatch, replace E by $\perp$ in the operand push it on E stack (WHY?)

Rename

Diagram:

$a_1 = \cdots$

$h_1 = \Phi(h_2, \bot)$
$a_2 = \phi(a_4, a_1)$

$a_2 + b_1 : [h_1]$
$a_3 = \cdots$

$h_2 = \Phi(h_1, \bot)$
$a_4 = \phi(a_2, a_3)$
$a_4 + b_1 : [h_2]$

EXIT

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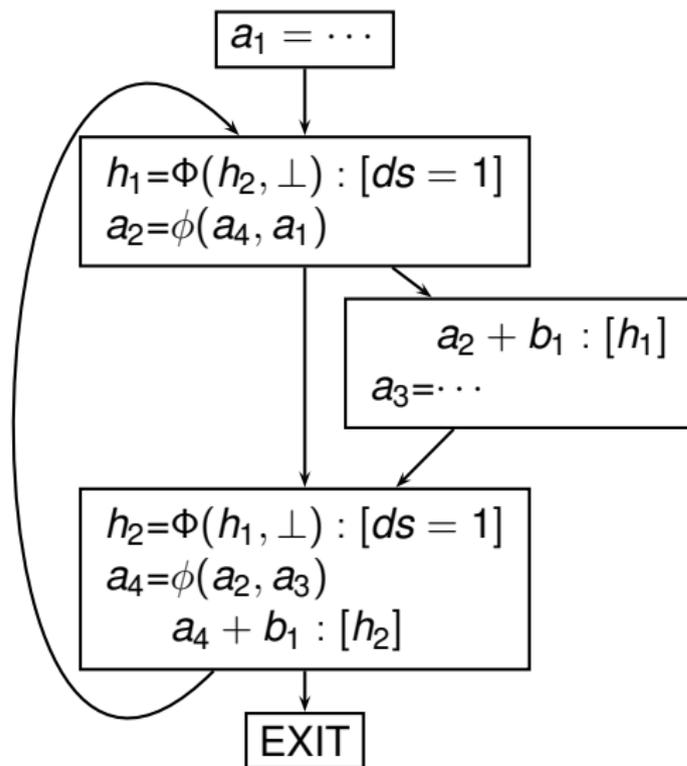- ▶ there is a path to EXIT from Φ along which the result of Φ is
  - ▶ either not used
  - ▶ used only as an operand of another Φ that itself is **NOT** down-safe
- ▶ *HasRealUse*: Real occurrence of an expression

Down-safety (ds = · · ·)


```
                    ┌──────────┐
                    │ a₁ = ··· │
                    └──────────┘
                         │
        ┌────────────────▼──────────────────┐
        │ h₁=Φ(h₂,⊥) : [ds = 1]             │
        │ a₂=φ(a₄, a₁)                       │
        └────────────────┬──────────────────┘
                         │          ┌──────────────────────┐
                         │          │         a₂ + b₁ : [h₁] │
                         │          │ a₃=···                 │
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        └────────────────┬──────────────────┘
                         │
                    ┌────▼─────┐
                    │  EXIT    │
                    └──────────┘
```

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WillBeAvail

- ▶ The set of Φs where the expression must be available in any computationally optimal placement
- ▶ Computation of *two forward* properties:
  - ▶ *CanBeAvail*: Φs for which E is either available or anticipable or both
  - ▶ *Later*: Φs beyond which insertion can not be postponed without introducing new redundancy

$$WillBeAvail = CanBeAvail \land \neg Later$$

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  - ▶ exclude edges along which *HasRealUse* is *true*

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- ▶ Later $\Rightarrow$ Φs that are CanBeAvail, but do not reach any real occurrence of *E*

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    - *WillBeAvail*(Φ) == *true*; AND
    - *Arg$_i$* is $\bot$; OR
        - (HasRealUse(*Arg$_i$*) == *false*), AND
        - *Arg$_i$* is defined by Φ' with *WillBeAvail*(Φ') == *false*

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  - If *WillBeAvail* is *true*, it is replaced by SSA temporary with appropriate version (h_x)

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- ▶ For each Φ for E
  - ▶ If *WillBeAvail* is *true*, it is replaced by SSA temporary with appropriate version (h_x)
  - ▶ If *WillBeAvail* is *false*, it is not part of SSA form, and is removed

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- ▶ AvailDef: Table to mark def of expression occurrences
- ▶ Computed for each class (say h_x) of E
- ▶ Preorder traversal of dominator tree

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        - ▶ Otherwise AvailDef[x] = this Φ (we must be visiting x for first time)

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        - ▶ Otherwise AvailDef[x] = this Φ (we must be visiting x for first time)

AvailDef Computation

- ▶ Initialize: AvailDef[x] = $\perp$ $\forall x$ (all classes of all expressions)
- ▶ During course of traversal, process occurrence x of E
  - ▶ Φ occurrence:
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        - ▶ If AvailDef[x] is $\perp$, mark this occurrence as def
        - ▶ Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def

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    - ▶ Else, mark this occurrence as use of AvailDef[x]

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    - ▶ Else, mark this occurrence as use of AvailDef[x]
  - ▶ Φ operand (processed in predecessor block P)

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        - ▶ Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
        - ▶ Else, mark this occurrence as use of AvailDef[x]
    - ▶ Φ operand (processed in predecessor block P)
   - ▶ If WillBeAvail of Φ is false, ignore.

AvailDef Computation

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    - ▶ Real occurrence:
        - ▶ If AvailDef[x] is $\perp$, mark this occurrence as def
        - ▶ Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
        - ▶ Else, mark this occurrence as use of AvailDef[x]
    - ▶ Φ operand (processed in predecessor block P)
        - ▶ If WillBeAvail of Φ is false, ignore.
        - ▶ Else, if *Insert* is true for the operand, insert computation of E in block P, set it as a def, mark this occurrence as use of inserted.

AvailDef Computation

- ▶ Initialize: AvailDef[x] = $\perp$ $\forall x$ (all classes of all expressions)
- ▶ During course of traversal, process occurrence x of E
  - ▶ Φ occurrence:
    - ▶ If WillBeAvail is *false*, ignore.
    - ▶ Otherwise AvailDef[x] = this Φ (we must be visiting x for first time) – WHY?
  - ▶ Real occurrence:
    - ▶ If AvailDef[x] is $\perp$, mark this occurrence as def
    - ▶ Else, if AvailDef[x] does not dominate this occurrence, mark this occurrence as def
    - ▶ Else, mark this occurrence as use of AvailDef[x]
  - ▶ Φ operand (processed in predecessor block P)
    - ▶ If WillBeAvail of Φ is false, ignore.
    - ▶ Else, if *Insert* is true for the operand, insert computation of E in block P, set it as a def, mark this occurrence as use of inserted.
    - ▶ Else (*Insert* is false), mark this occurrence as use of AvailDef[x]

Finalize
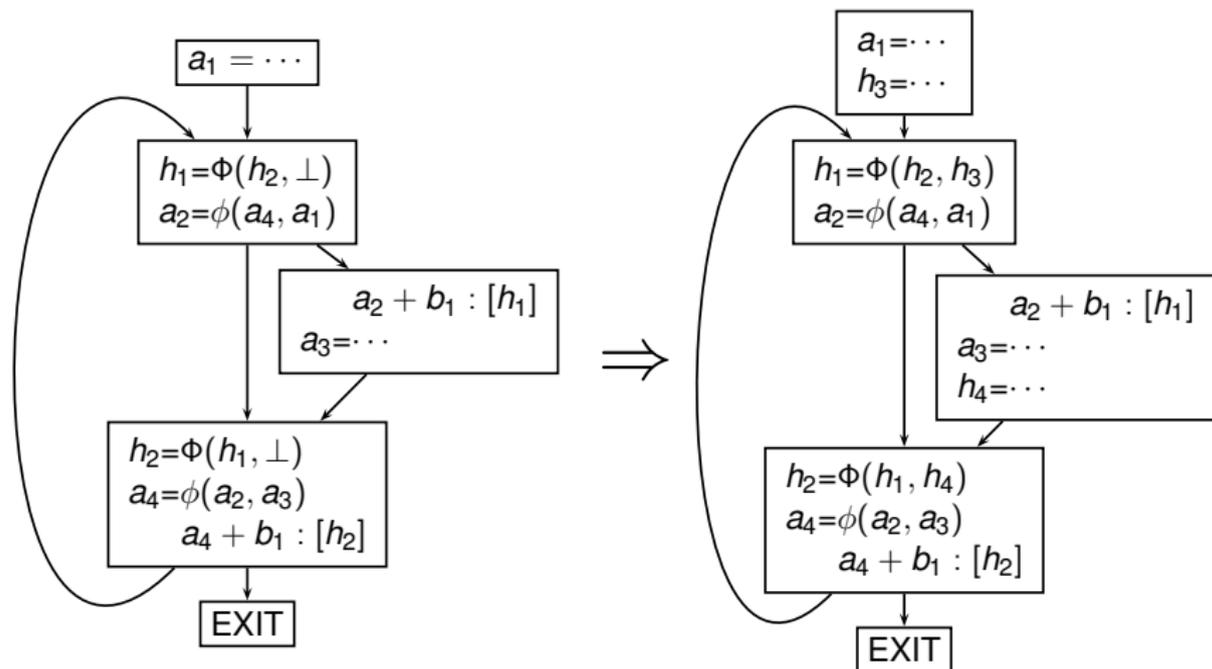
Code Motion

- For real *def* occurrence of E, compute E in a new version of temporary t

Code Motion

- For real *def* occurrence of E, compute E in a new version of temporary t
- For real *use* occurrence of E, replace E by current version of t

Code Motion

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- For real *use* occurrence of E, replace E by current version of t
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Code Motion

- For real *def* occurrence of E, compute E in a new version of temporary t
- For real *use* occurrence of E, replace E by current version of t
- For inserted occurrence of E, compute E in a new version of temporary t
- For a Φ occurrence, insert appropriate ϕ for t

Code Motion

Left diagram:

```
a₁=···
h₃=···
```
$$a_1 = \cdots$$
$$h_3 = \cdots$$

$$h_1 = \Phi(h_2, h_3)$$
$$a_2 = \phi(a_4, a_1)$$

$$a_2 + b_1 : [h_1]$$
$$a_3 = \cdots$$
$$h_4 = \cdots$$

$$h_2 = \Phi(h_1, h_4)$$
$$a_4 = \phi(a_2, a_3)$$
$$a_4 + b_1 : [h_2]$$

EXIT

$\Longrightarrow$

Right diagram:

$$a_1 = \cdots$$
$$t_1 = a_1 + b_1$$

$$t_2 = \phi(t_4, t_1)$$
$$a_2 = \phi(a_4, a_1)$$

$$t_2$$
$$a_3 = \cdots$$
$$t_3 = a_3 + b_1$$

$$t_4 = \phi(t_2, t_3)$$
$$a_4 = \phi(a_2, a_3)$$
$$t_4$$

EXIT