CS738: Advanced Compiler Optimizations

# Data Flow Analysis

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## Agenda

- Static analysis and compile-time optimizations
- For the next few lectures
- *Intraprocedural* Data Flow Analysis
  - Classical Examples
  - Components

## Assumptions

- Intraprocedural: Restricted to a single function
- Input in 3-address format
- Unless otherwise specified

## 3-address Code Format

- Assignments
  - x = y op z
  - x = op y
  - x = y
- Jump/control transfer
  - goto L
  - if x relop y goto L
- Statements can have label(s)
  - L: . . .
- Arrays, Pointers and Functions to be added later when needed

## Data Flow Analysis

- Class of techniques to derive information about flow of data
  - along program execution paths
- Used to answer questions such as:
  - whether two identical expressions evaluate to same value
    - used in common subexpression elimination
  - whether the result of an assignment is used later
    - used by dead code elimination

## Data Flow Abstraction

- Basic Blocks (BB)
  - sequence of 3-address code stmts
  - single entry at the first statement
  - single exit at the last statement
  - Typically we use "maximal" basic block (maximal sequence of such instructions)

## Identifying Basic Blocks

- *Leader*: The first statement of a basic block
  - The first instruction of the program (procedure)
  - Target of a branch (conditional and unconditional goto)
  - Instruction immediately following a branch

## Special Basic Blocks

- Two special BBs are added to simplify the analysis
  - empty (?) blocks!
- *Entry*: The first block to be executed for the procedure analyzed
- *Exit*: The last block to be executed

## Data Flow Abstraction

- Control Flow Graph (CFG)
- A rooted directed graph $G = (N, E)$
- $N$ = set of BBs
  - including *Entry*, *Exit*
- $E$ = set of edges

## CFG Edges

- Edge $B_1 \rightarrow B_2 \in E$ if control can transfer from $B_1$ to $B_2$
  - Fall through
  - Through jump (goto)
  - Edge from *Entry* to (all?) real first BB(s)
  - Edge to *Exit* from all last BBs
    - BBs containing return
    - Last real BB

## Data Flow Abstraction: Control Flow Graph

- Graph representation of paths that program may exercise during execution
- Typically one graph per procedure
- Graphs for separate procedures have to be combined/connected for interprocedural analysis
  - Later!
  - Single procedure, single flow graph for now.

## Data Flow Abstraction: Program Points

- Input state/Output state for Stmt
  - Program point before/after a stmt
  - Denoted IN[s] and OUT[s]
  - Within a basic block:
    - Program point after a stmt is same as the program point before the next stmt

## Data Flow Abstraction: Program Points

- Input state/Output state for BBs
  - Program point before/after a bb
  - Denoted IN[$B$] and OUT[$B$]
  - For $B_1$ and $B_2$:
    - if there is an edge from $B_1$ to $B_2$ in CFG, then the program point *after* the last stmt of $B_1$ *may be* followed immediately by the program point *before* the first stmt of $B_2$.

## Data Flow Abstraction: Execution Paths

- An execution path is of the form

$$p_1, p_2, p_3, \ldots, p_n$$

  where $p_i \rightarrow p_{i+1}$ are adjacent program points in the CFG.
- Infinite number of possible execution paths in practical programs.
- Paths having no finite upper bound on the length.
- Need to *summarize* the information at a program point with a finite set of facts.

## Data Flow Schema

- Data flow values associated with each program point
  - Summarize all possible states at that point
- *Domain:* set of all possible data flow values
- Different domains for different analyses/optimizations

## Data Flow Problem

- Constraints on data flow values
  - Transfer constraints
  - Control flow constraints
- **Aim:** To find a solution to the constraints
  - Multiple solutions possible
  - Trivial solutions, ..., Exact solutions
- We typically compute approximate solution
  - Close to the exact solution (as close as possible!)
  - Why not exact solution?

## Data Flow Constraints: Transfer Constraints

- Transfer functions
  - relationship between the data flow values before and after a stmt
- forward functions: Compute facts *after* a statement $s$ from the facts available *before* $s$.
  - General form:
    $$\text{OUT}[s] = f_s(\text{IN}[s])$$
- backward functions: Compute facts *before* a statement $s$ from the facts available *after* $s$.
  - General form:
    $$\text{IN}[s] = f_s(\text{OUT}[s])$$
- $f_s$ depends on the statement and the analysis

## Data Flow Constraints: Control Flow Constraints

- Relationship between the data flow values of two points that are related by program execution semantics
- For a basic block having $n$ statements:
  $$\text{IN}[s_{i+1}] = \text{OUT}[s_i], i = 1, 2, \ldots, n-1$$
- $\text{IN}[s_1]$, $\text{OUT}[s_n]$ to come later

## Data Flow Constraints: Notations

- PRED (B): Set of predecessor BBs of block $B$ in CFG
- SUCC (B): Set of successor BBs of block $B$ in CFG
- $f \circ g$ : Composition of functions $f$ and $g$
- $\bigoplus$: An abstract operator denoting some way of combining facts present in a set .

## Data Flow Constraints: Basic Blocks

- **Forward**
  - For $B$ consisting of $s_1, s_2, \ldots, s_n$
    $$f_B = f_{s_n} \circ \ldots \circ f_{s_2} \circ f_{s_1}$$
    $$\text{OUT}[B] = f_B(\text{IN}[B])$$
  - Control flow constraints
    $$\text{IN}[B] = \bigoplus_{P \in \text{PRED}(B)} \text{OUT}[P]$$
- **Backward**
  $$f_B = f_{s_1} \circ f_{s_2} \circ \ldots \circ f_{s_n}$$
  $$\text{IN}[B] = f_B(OUT[B])$$
  $$\text{OUT}[B] = \bigoplus_{S \in \text{SUCC}(B)} IN[S]$$

## Data Flow Equations

- Typical Equation

$$\text{OUT}[s] = \text{IN}[s] - \textit{kill}[s] \cup \textit{gen}[s]$$

  *gen*($s$): information generated

  *kill*($s$): information killed

- Example:

```
a  =  b*c  // generates expression b * c
c  =  5    // kills expression b*c
d  =  b*c  // is b*c redundant here?
```

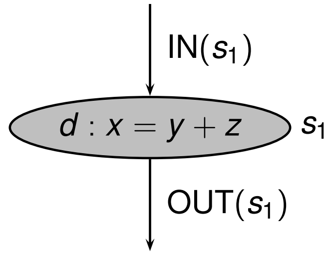
## Example Data Flow Analysis

- Reaching Definitions Analysis
- Definition of a variable $x$: $x = \ldots$something$\ldots$
- Could be more complex (e.g. through pointers, references, implicit)

## Reaching Definitions Analysis

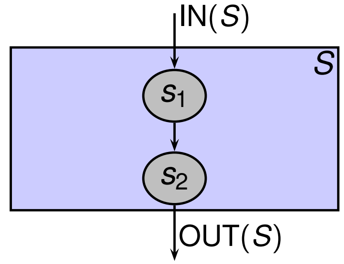
- A definition $d$ reaches a point $p$ if
  - there is a path from the point *immediately following* $d$ to $p$
  - $d$ is not "killed" along that path
  - "Kill" means redefinition of the left hand side ($x$ in the earlier example)

## RD Analysis of a Structured Program



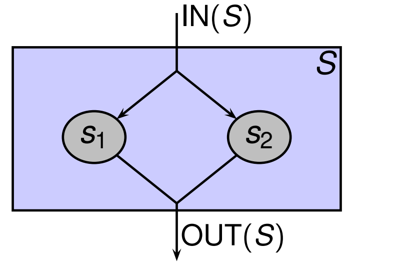

$$
\begin{aligned}
\text{OUT}(s_1) &= \text{IN}(s_1) - \text{KILL}(s_1) \cup \text{GEN}(s_1) \\
\text{GEN}(s_1) &= \{d\} \\
\text{KILL}(s_1) &= D_x - \{d\}, \text{where } D_x\text{: set of all definitions of } x \\
\text{KILL}(s_1) &= D_x\text{? will also work here} \\
&\quad\ \text{but may not work in general}
\end{aligned}
$$

## RD Analysis of a Structured Program



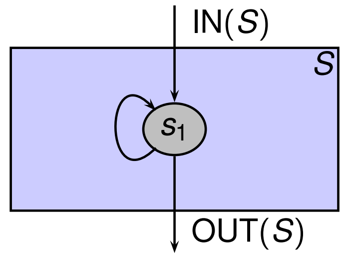

$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) - \mathrm{KILL}(s_2) \cup \mathrm{GEN}(s_2)\\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) - \mathrm{GEN}(s_2) \cup \mathrm{KILL}(s_2)\\
\mathrm{IN}(s_1) &= \mathrm{IN}(S)\\
\mathrm{IN}(s_2) &= \mathrm{OUT}(s_1)\\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_2)
\end{aligned}
$$

## RD Analysis of a Structured Program



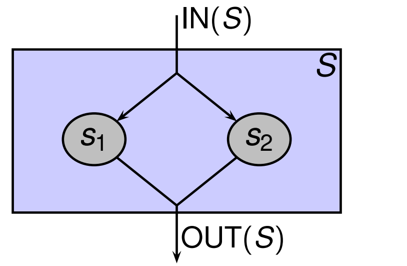

$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \cup \mathrm{GEN}(s_2)\\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \cap \mathrm{KILL}(s_2)\\
\mathrm{IN}(s_1) &= \mathrm{IN}(s_2) = \mathrm{IN}(S)\\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_1) \cup \mathrm{OUT}(s_2)
\end{aligned}
$$

## RD Analysis of a Structured Program




$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1)\\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1)\\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_1)\\
\mathrm{IN}(s_1) &= \mathrm{IN}(S) \cup \mathrm{GEN}(s_1)
\end{aligned}
$$

## RD Analysis is Approximate

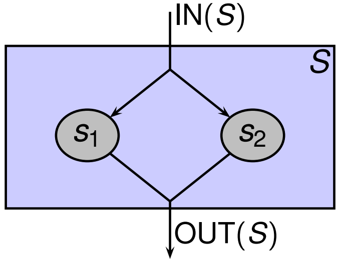



- Assumption: All paths are feasible.
- Example:

```
if (true) s1;
else      s2;
```

| Fact | | Computed | | Actual |
|---|---|---|---|---|
| $\mathrm{GEN}(S)$ | $=$ | $\mathrm{GEN}(s_1) \cup \mathrm{GEN}(s_2)$ | $\supseteq$ | $\mathrm{GEN}(s_1)$ |
| $\mathrm{KILL}(S)$ | $=$ | $\mathrm{KILL}(s_1) \cap \mathrm{KILL}(s_2)$ | $\subseteq$ | $\mathrm{KILL}(s_1)$ |

## RD Analysis is Approximate




- Thus,

  true GEN($S$) $\subseteq$ analysis GEN($S$)
  true KILL($S$) $\supseteq$ analysis KILL($S$)
- More definitions computed to be reaching than actually do!
- Later we shall see that this is SAFE approximation
  - prevents optimizations
  - but NO wrong optimization

## RD at BB level

- A definition $d$ can reach the start of a block from any of its predecessor
  - if it reaches the end of some predecessor

$$\text{IN}(B) = \bigcup_{P \in \text{PRED}(B)} \text{OUT}(P)$$

- A definition $d$ reaches the end of a block if
  - either it is generated in the block
  - or it reaches block and not killed

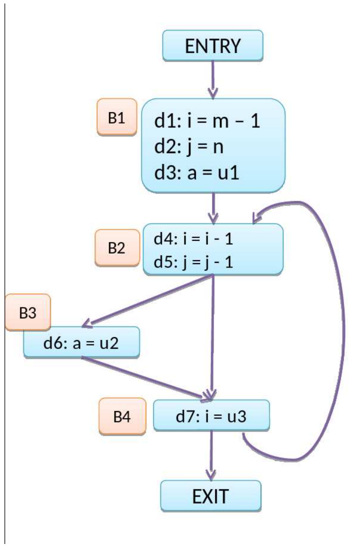
$$\text{OUT}(B) = \text{IN}(B) - \text{KILL}(B) \cup \text{GEN}(B)$$

## Solving RD Constraints

- KILL & GEN known for each BB.
- A program with $N$ BBs has $2N$ equations with $2N$ unknowns.
  - Solution is possible.
  - Iterative approach (on the next slide).

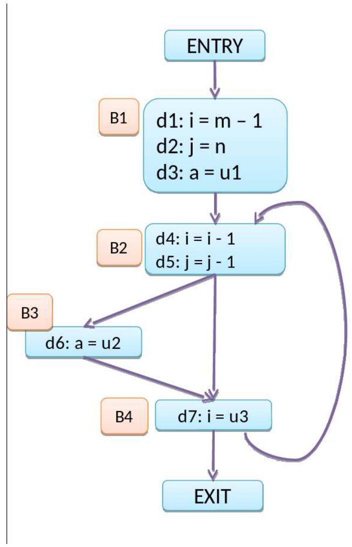
```
for each block B {
    OUT(B) = ∅;
}
OUT(Entry) = ∅; // note this for later discussion
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        IN(B) = ⋃_{P∈PRED(B)} OUT(P);
        oldOut = OUT(B);
        OUT(B) = IN(B) − KILL(B) ∪ GEN(B);
        if (OUT(B) ≠oldOut) then {
            change = true;
        }
    }
}
```

## Reaching Definitions: Example



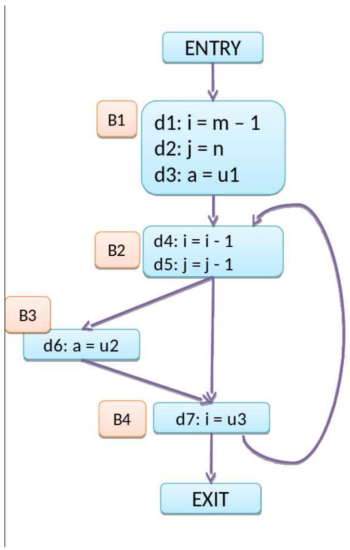

| BB | GEN | KILL |
|---|---|---|
| B1 | {d1, d2, d3} | {d4, d5, d6, d7} |
| B2 | {d4, d5} | {d1, d2, d7} |
| B3 | {d6} | {d3} |
| B4 | {d7} | {d1, d4} |

## Reaching Definitions: Example




| Pass# | Pt | B1 | B2 | B3 | B4 |
|---|---|---|---|---|---|
| Init | IN | - | - | - | - |
| | OUT | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| 1 | IN | $\emptyset$ | d1, d2, d3 | d3, d4, d5 | d3, d4, d5, d6 |
| | OUT | d1, d2, d3 | d3, d4, d5 | d4, d5, d6 | d3, d5, d6, d7 |
| 2 | IN | $\emptyset$ | d1, d2, d3, d5, d6, d7 | d3, d4, d5, d6 | d3, d4, d5, d6 |
| | OUT | d1, d2, d3 | d3, d4, d5, d6 | d4, d5, d6 | d3, d5, d6, d7 |
| 3 | IN | $\emptyset$ | d1, d2, d3, d5, d6, d7 | d3, d4, d5, d6 | d3, d4, d5, d6 |
| | OUT | d1, d2, d3 | d3, d4, d5, d6 | d4, d5, d6 | d3, d5, d6, d7 |

## Reaching Definitions: Bitvectors




a bit for each definition:

| | | | | | | |
|---|---|---|---|---|---|---|
| d1 | d2 | d3 | d4 | d5 | d6 | d7 |

| Pass# | Pt | B1 | B2 | B3 | B4 |
|---|---|---|---|---|---|
| Init | IN | - | - | - | - |
| | OUT | 0000000 | 0000000 | 0000000 | 0000000 |
| 1 | IN | 0000000 | 1110000 | 0011100 | 0011110 |
| | OUT | 1110000 | 0011100 | 0001110 | 0010111 |
| 2 | IN | 0000000 | 1110111 | 0011110 | 0011110 |
| | OUT | 1110000 | 0011110 | 0001110 | 0010111 |
| 3 | IN | 0000000 | 1110111 | 0011110 | 0011110 |
| | OUT | 1110000 | 0011110 | 0001110 | 0010111 |

## Reaching Definitions: Bitvectors

- Set-theoretic definitions:

$$\mathrm{IN}(B) = \bigcup_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$$

$$\mathrm{OUT}(B) = \mathrm{IN}(B) - \mathrm{KILL}(B) \cup \mathrm{GEN}(B)$$

- Bitvector definitions:

$$\mathrm{IN}(B) = \bigvee_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$$

$$\mathrm{OUT}(B) = \mathrm{IN}(B) \wedge \neg\mathrm{KILL}(B) \vee \mathrm{GEN}(B)$$

- Bitwise $\vee, \wedge, \neg$ operators

## Reaching Definitions: Application

**Constant Folding**

```
while changes occur {
   forall the stmts S of the program {
      foreach operand B of S {
         if there is a unique definition of B
         that reaches S and is a constant C {
            replace B by C in S;
            if all operands of S are constant {
               replace rhs by eval(rhs);
               mark definition as constant;
}}}}}
```

## Reaching Definitions: Application

- Recall the approximation in reaching definition analysis
  true GEN($S$) $\subseteq$ analysis GEN($S$)
  true KILL($S$) $\supseteq$ analysis KILL($S$)
- Can it cause the application to infer
  - an expression as a constant when it is has different values for different executions?
  - an expression as not a constant when it is a constant for all executions?
- Safety? Profitability?

## Reaching Definitions: Summary

- $\text{GEN}(B) = \left\{ d_x \;\middle|\; \begin{array}{l} d_x \text{ in } B \text{ defines variable } x \text{ and is not} \\ \text{followed by another definition of } x \text{ in } B \end{array} \right\}$
- $\text{KILL}(B) = \{d_x \mid B \text{ contains some definition of } x\}$
- $\text{IN}(B) = \bigcup_{P \in \text{PRED}(B)} \text{OUT}(P)$
- $\text{OUT}(B) = \text{IN}(B) - \text{KILL}(B) \cup \text{GEN}(B)$
- meet ( $\bigwedge$ ) operator: The operator to combine information coming along different predecessors is $\cup$
- What about the *Entry* block?

## Reaching Definitions: Summary

- Entry block has to be initialized specially:

$$\begin{aligned} \text{OUT}(\mathit{Entry}) &= \text{EntryInfo} \\ \text{EntryInfo} &= \emptyset \end{aligned}$$

- A better entry info could be:

$$\text{EntryInfo} = \{x = \mathit{undefined} \mid x \mathit{\ is\ a\ variable}\}$$

- Why?