Code Generation: Sethi Ullman Algorithm

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- Complexity is linear in the size of the expression tree. Reasonably efficient.

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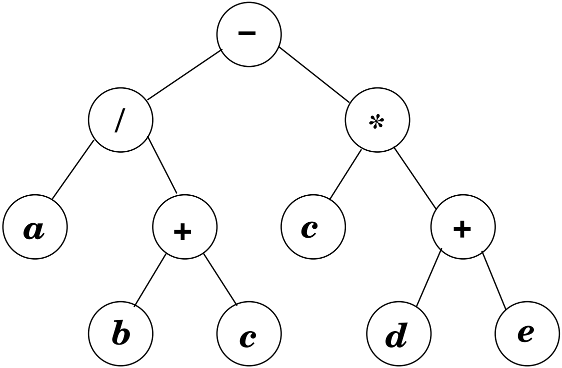
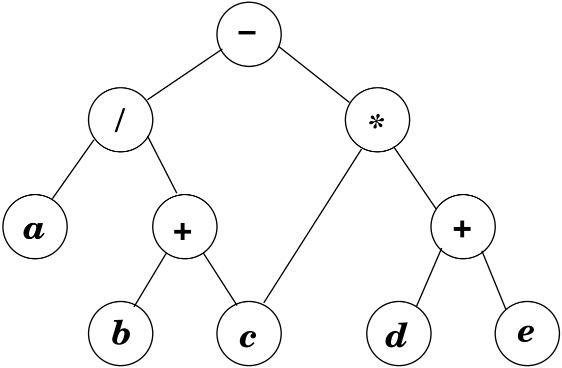


Expression Trees

- We have not identified common sub-expressions; else we would have a directed acyclic graph (DAG):



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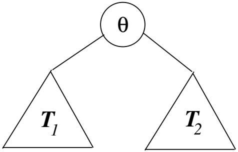
     then 

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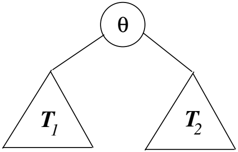
     then  is an expression tree.

- In this example

  $\Sigma = \{a, b, c, d, e, \ldots\}$, and $\Theta = \{+, -, *, /, \ldots\}$

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- Note:
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  2. In instruction 4, the destination register is the same as the left operand register.

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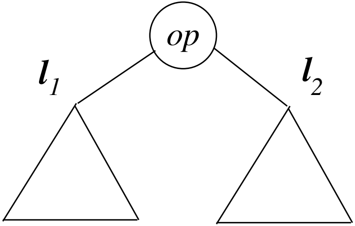
- If the left and right subtrees require l_1, and l_2 ($l_1 < l_2$) registers respectively, what should be the order of evaluation?



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- The maximum register requirement in this case is $max(l_1, l_2 + 1) = l_2 + 1$.

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Therefore the subtree requiring more registers should be evaluated first.

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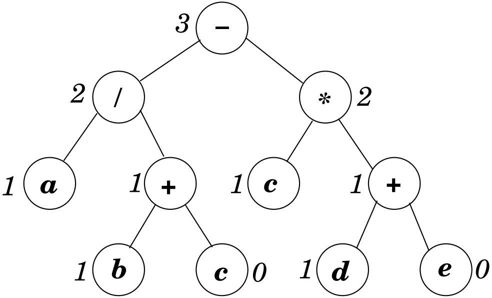

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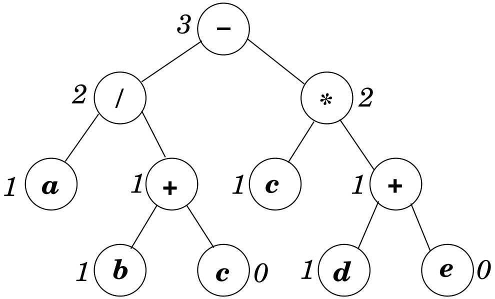

- Left and the right leaves are labeled 1 and 0 respectively, because the left leaf must necessarily be in a register, whereas the right leaf can reside in memory.

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- Visit the tree in post-order. For every node visited do:
  1. Label each left leaf by 1 and each right leaf by 0.
  2. If the labels of the children of a node n are l_1 and l_2 respectively, then

$$\begin{aligned} label(n) &= max(l_1, l_2), if\, l_1 \neq l_2 \\ &= l_1 + 1, otherwise \end{aligned}$$

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5. *swap*(*rstack*) swaps the top two registers on the stack.

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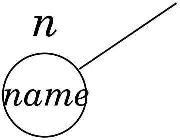
  1. *n is a left leaf:*



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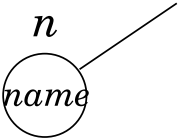

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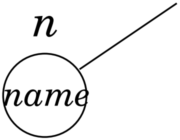
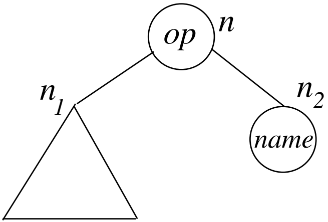


$$gen(top(rstack) \leftarrow name)$$

Comments: *n* is named by a variable say *name*. Code is generated to load *name* into a register.

The Algorithm

2. *n's right child is a leaf:*



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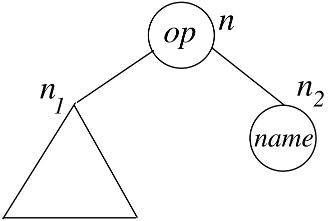

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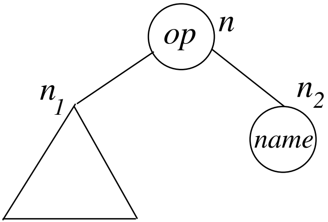

$gencode(n_1)$

$gen(top(rstack) \leftarrow top(rstack)\ op\ name)$

Comments: n_1 is first evaluated in the register on the top of the stack, followed by the operation *op* leaving the result in the same register.

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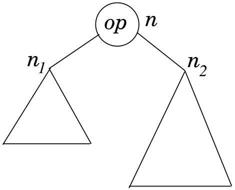
3. *The left child of n requires lesser number of registers. This requirement is strictly less than the available number of registers*



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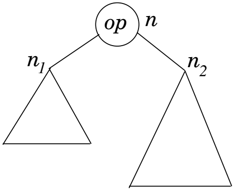

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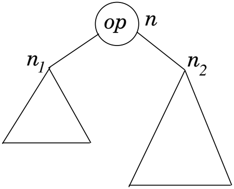

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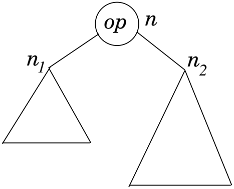

swap(*rstack*);	Right child goes into next to top register
gencode(n_2);	Evaluate right child

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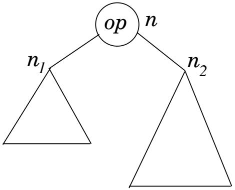

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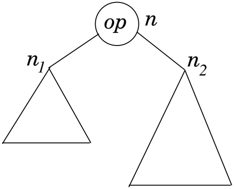

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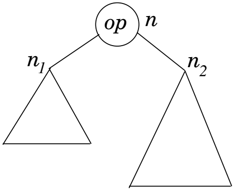

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gen(*top*(*rstack*) ← *top*(*rstack*) *op* R); Issue *op*

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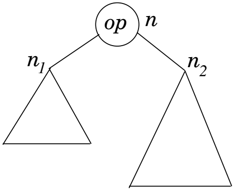

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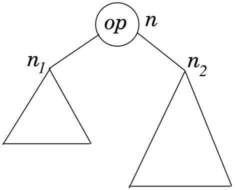

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$swap(rstack)$	Restore register stack

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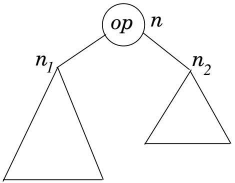

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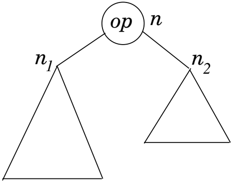

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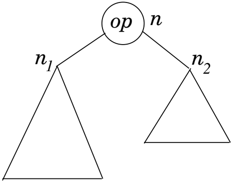

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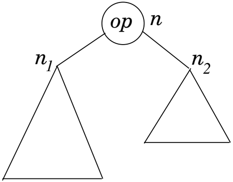

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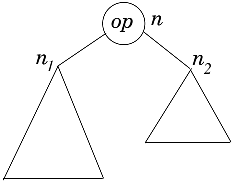

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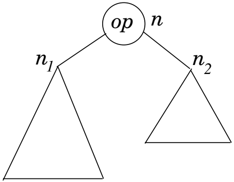

$gencode(n_1);$

$R := pop(rstack);$

$gencode(n_2);$

$gen(R \leftarrow R\ op\ top(rstack));$

The Algorithm

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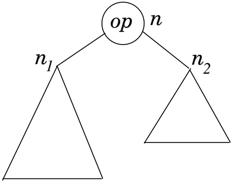

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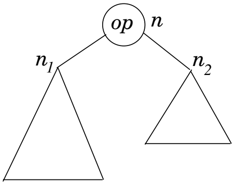

$gencode(n_1);$
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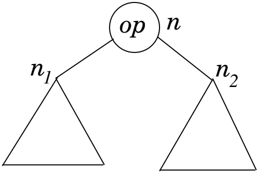
$push(rstack, R)$

Comments: Same as case 3, except that the left sub-tree is evaluated first.

The Algorithm

5. *Both the children of n require registers greater or equal to the available number of registers.*



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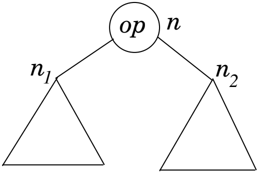

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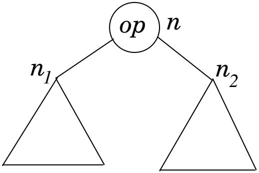

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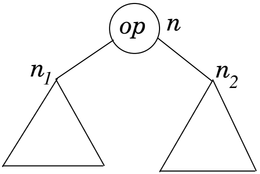

$gencode(n_2)$;

$T := pop(tstack)$;

The Algorithm

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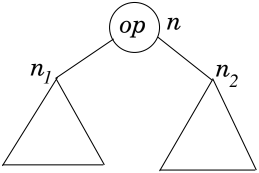

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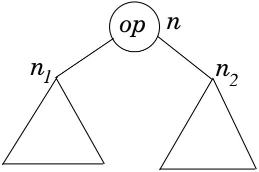

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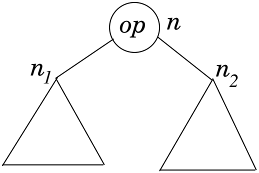

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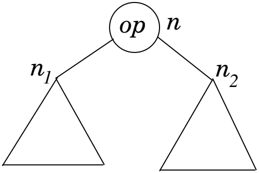

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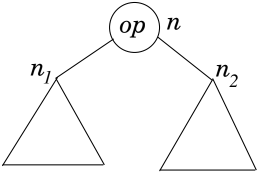

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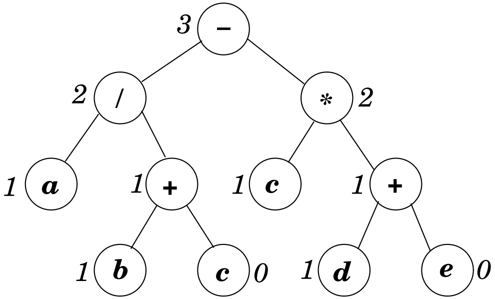
Comments: In this case the right sub-tree is first evaluated into a temporary. This is followed by the evaluations of the left sub-tree and *n* into the register on the top of the stack.

An Example

For the example:



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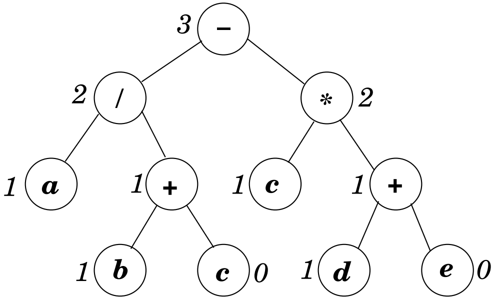
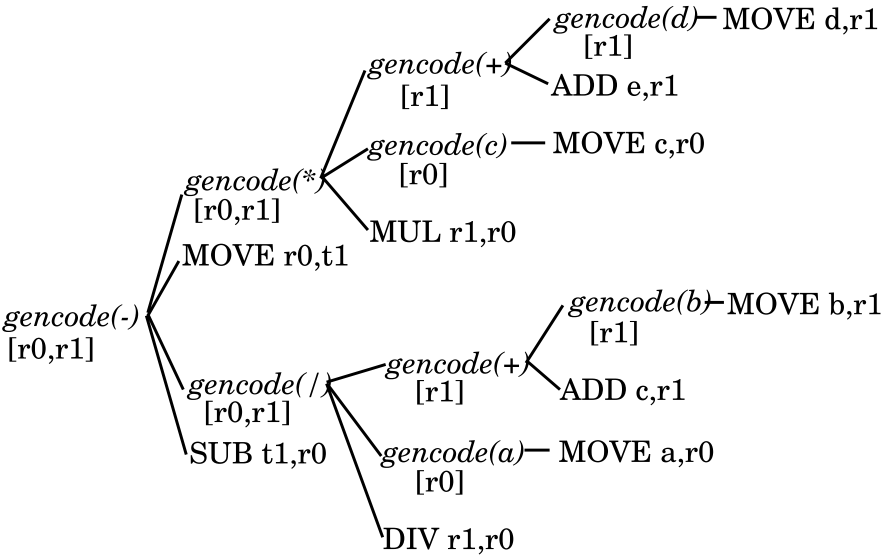


assuming two available registers r_0 and r_1, the calls to gencode and the generated code are shown on the next slide.

An Example


gencode(-)
[r0,r1]
gencode(*)
[r0,r1]
MOVE r0,t1
gencode(/)
[r0,r1]
SUB t1,r0
gencode(+)
[r1]
gencode(c)
[r0]
MUL r1,r0
gencode(d)
[r1]
MOVE d,r1
ADD e,r1
MOVE c,r0
gencode(+)
[r1]
gencode(a)
[r0]
DIV r1,r0
gencode(b)
[r1]
MOVE b,r1
ADD c,r1
MOVE a,r0

SETHI-ULLMAN ALGORITHM: OPTIMALITY

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- We shall now elaborate on each of these.

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2. Each node of the expression tree is visited exactly once. If this node specifies a binary operation, then the algorithm branches into steps 2,3,4 or 5, and at each of these places code is generated to perform this operation exactly once.

SETHI-ULLMAN ALGORITHM: OPTIMALITY

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3. The number of stores is optimal: this is harder to show.
   - Define a *major node* as a node, each of whose children has a label at least equal to the number of available registers.
   - If we can show that the number of stores required by *any program* computing an expression tree is at least equal the number of major nodes, then our algorithm produces minimal number of stores (Why?)

SETHI-ULLMAN ALGORITHM

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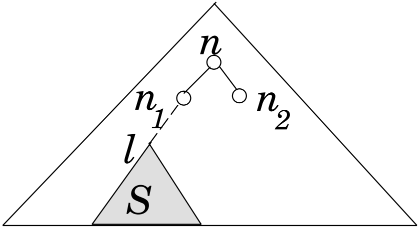

- Let n be the major node in the original tree, just above S, and n_1 and n_2 be its immediate descendants (n_1 could be l itself).

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3. The node n may no longer be a major node *but all other major nodes in the original tree continue to be major nodes in the modified tree*.
4. Therefore the number of major nodes in the modified tree is $M - 1$.
5. If we assume as induction hypothesis that the number of stores for the modified tree is at least $M - 1$, then the number of stores for the original tree is at least M.

SETHI-ULLMAN ALGORITHM: COMPLEXITY

Since the algorithm visits every node of the expression tree twice – once during labeling, and once during code generation, the complexity of the algorithm is $O(n)$.