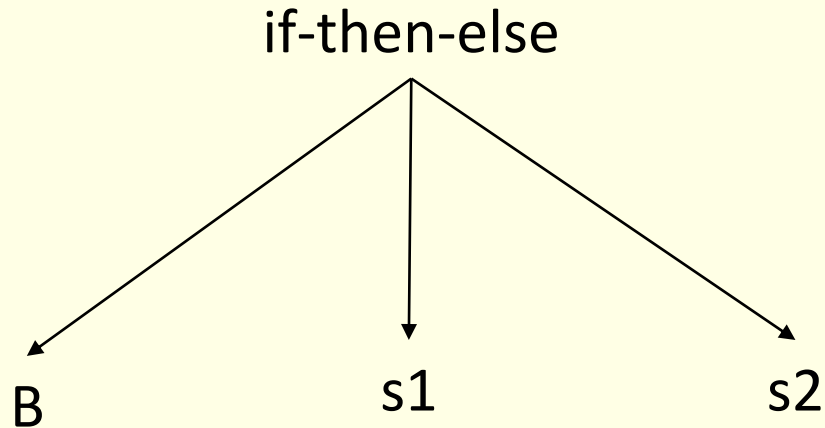


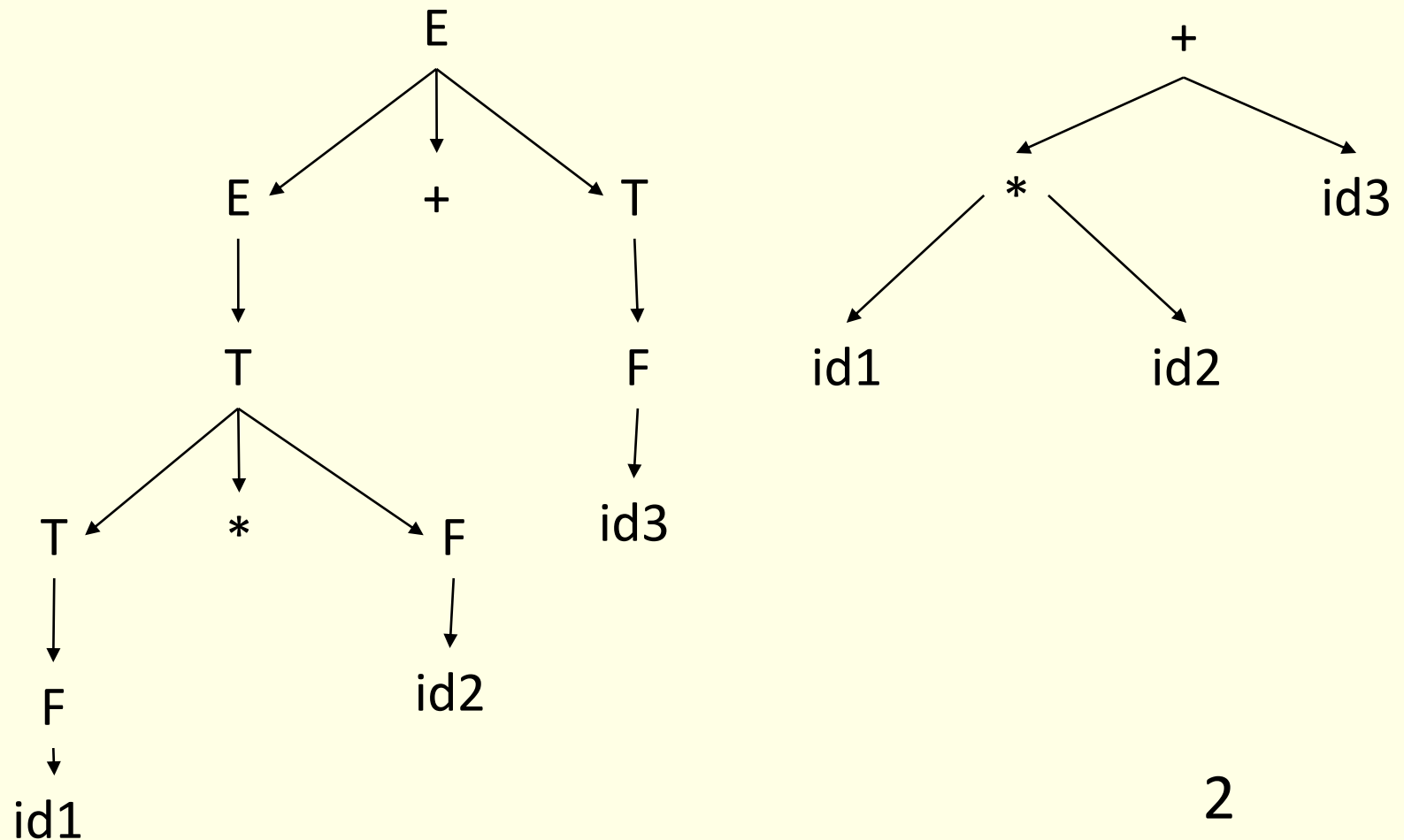
# Abstract Syntax Tree

- Condensed form of parse tree,
- useful for representing language constructs.
- The production  $S \rightarrow \text{if } B \text{ then } s1 \text{ else } s2$  may appear as



# Abstract Syntax tree ...

- Chain of single productions may be collapsed, and operators move to the parent nodes



# Constructing Abstract Syntax Tree for expression

- Each node can be represented as a record
- *operators*: one field for operator, remaining fields ptrs to operands  
    mknode(op,left,right )
- *identifier*: one field with label id and another ptr to symbol table  
    mkleaf(id,entry)
- *number*: one field with label num and another to keep the value of the number  
    mkleaf(num,val)

# Example

the following  
sequence of function  
calls creates a parse  
tree for  $a - 4 + c$

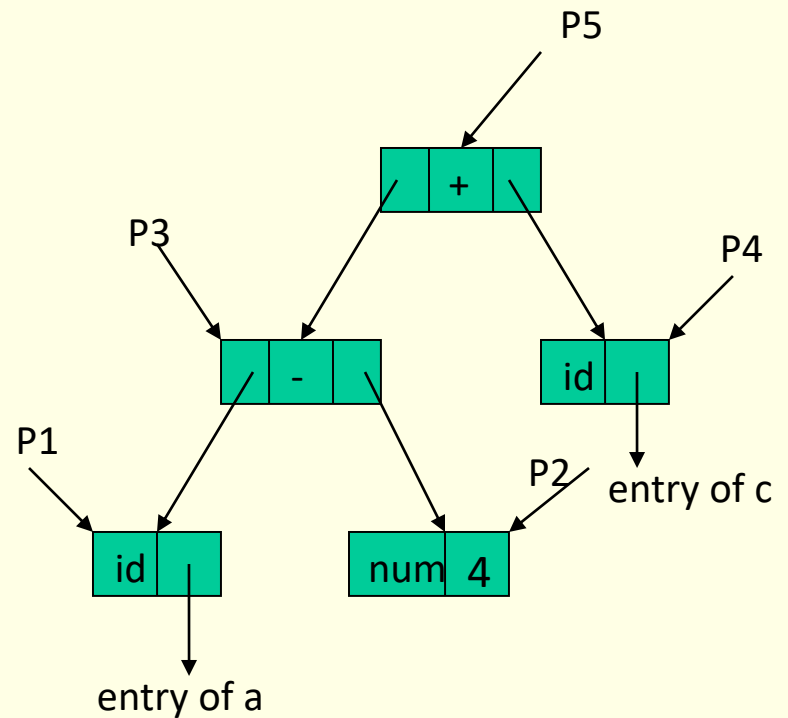
$P_1 = \text{mkleaf}(\text{id}, \text{entry.a})$

$P_2 = \text{mkleaf}(\text{num}, 4)$

$P_3 = \text{mknnode}(-, P_1, P_2)$

$P_4 = \text{mkleaf}(\text{id}, \text{entry.c})$

$P_5 = \text{mknnode}(+, P_3, P_4)$



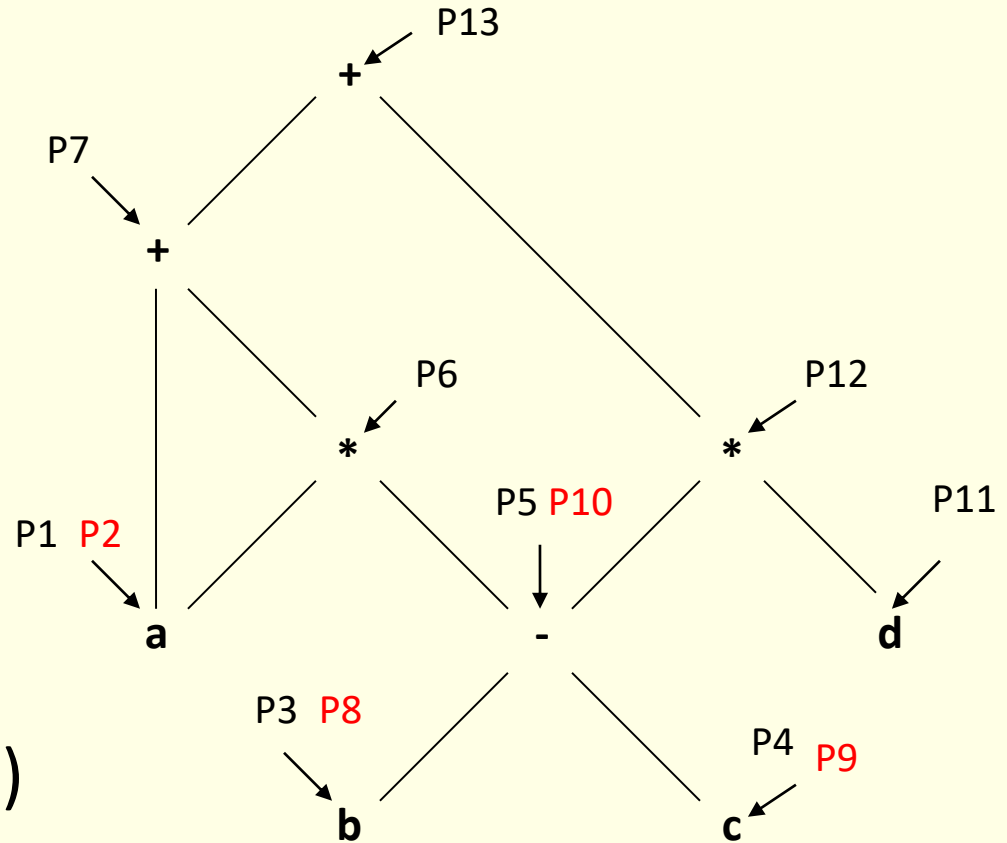
# A syntax directed definition for constructing syntax tree

$E \rightarrow E_1 + T$	$E.\text{ptr} = \text{mknode}(+, E_1.\text{ptr}, T.\text{ptr})$
$E \rightarrow T$	$E.\text{ptr} = T.\text{ptr}$
$T \rightarrow T_1 * F$	$T.\text{ptr} := \text{mknode}(*, T_1.\text{ptr}, F.\text{ptr})$
$T \rightarrow F$	$T.\text{ptr} := F.\text{ptr}$
$F \rightarrow (E)$	$F.\text{ptr} := E.\text{ptr}$
$F \rightarrow \text{id}$	$F.\text{ptr} := \text{mkleaf}(\text{id}, \text{entry.id})$
$F \rightarrow \text{num}$	$F.\text{ptr} := \text{mkleaf}(\text{num}, \text{val})$

# DAG for Expressions

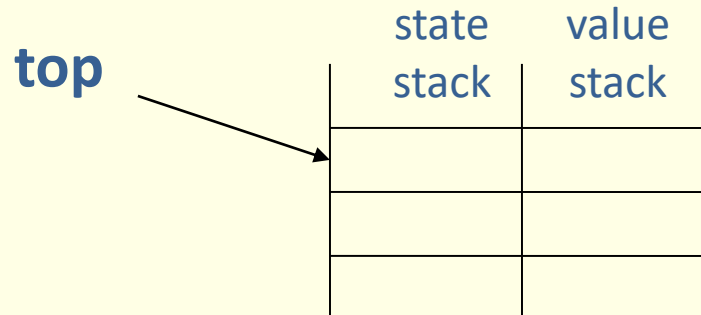
Expression  $a + a * (b - c) + (b - c) * d$   
make a leaf or node if not present,  
otherwise return pointer to the existing node

$P_1 = \text{makeleaf}(\text{id}, a)$   
 $P_2 = \text{makeleaf}(\text{id}, a)$   
 $P_3 = \text{makeleaf}(\text{id}, b)$   
 $P_4 = \text{makeleaf}(\text{id}, c)$   
 $P_5 = \text{makenode}(-, P_3, P_4)$   
 $P_6 = \text{makenode}(*, P_2, P_5)$   
 $P_7 = \text{makenode}(+, P_1, P_6)$   
 $P_8 = \text{makeleaf}(\text{id}, b)$   
 $P_9 = \text{makeleaf}(\text{id}, c)$   
 $P_{10} = \text{makenode}(-, P_8, P_9)$   
 $P_{11} = \text{makeleaf}(\text{id}, d)$   
 $P_{12} = \text{makenode}(*, P_{10}, P_{11})$   
 $P_{13} = \text{makenode}(+, P_7, P_{12})$



# Bottom-up evaluation of S-attributed definitions

- Can be evaluated while parsing
- Whenever reduction is made, value of new synthesized attribute is computed from the attributes on the stack
- Extend stack to hold the values also
- The current top of stack is indicated by **top** pointer



# Bottom-up evaluation of S-attributed definitions

- Suppose semantic rule
$$A.a = f(X.x, Y.y, Z.z)$$
is associated with production
$$A \rightarrow XYZ$$
- Before reducing  $XYZ$  to  $A$ , value of  $Z$  is in  $val(top)$ , value of  $Y$  is in  $val(top-1)$  and value of  $X$  is in  $val(top-2)$
- If symbol has no attribute then the entry is undefined
- After the reduction,  $top$  is decremented by 2 and state covering  $A$  is put in  $val(top)$



# Example: desk calculator

$L \rightarrow E \$$	Print (E.val)
$E \rightarrow E + T$	E.val = E.val + T.val
$E \rightarrow T$	E.val = T.val
$T \rightarrow T * F$	T.val = T.val * F.val
$T \rightarrow F$	T.val = F.val
$F \rightarrow (E)$	F.val = E.val
$F \rightarrow \text{digit}$	F.val = digit.lexval

# Example: desk calculator

$L \rightarrow E\$$

$E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow \text{digit}$

Before reduction  $ntop = top - r + 1$

After code reduction  $top = ntop$

$r$  is the #symbols on RHS

INPUT	STATE	Val	PROD
3*5+4\$			
*5+4\$	digit	3	
*5+4\$	F	3	F → digit
*5+4\$	T	3	T → F
5+4\$	T*	3 □	
+4\$	T* digit	3 □ 5	
+4\$	T* F	3 □ 5	F → digit
+4\$	T	15	T → T * F
+4\$	E	15	E → T
4\$	E+	15 □	
\$	E+ digit	15 □ 4	
\$	E+ F	15 □ 4	F → digit
\$	E+ T	15 □ 4	T → F
\$	E	19	E → E + T

# YACC Terminology

$E \rightarrow E + T$      $\text{val}(\text{ntop}) = \text{val}(\text{top}-2) + \text{val}(\text{top})$

In YACC

$E \rightarrow E + T$      $\$\$ = \$1 + \$3$

$\$\$$  maps to  $\text{val}[\text{top} - r + 1]$

$\$k$  maps to  $\text{val}[\text{top} - r + k]$

$r = \#$ symbols on RHS ( here 3)

$\$\$ = \$1$  is the *default* action in YACC

# L-attributed definitions

- When translation takes place during parsing, order of evaluation is linked to the order in which nodes are created
- In S-attributed definitions parent's attribute evaluated after child's.
- A natural order in both top-down and bottom-up parsing is depth first-order
- **L-attributed** definition: where attributes can be evaluated in depth-first order

# L attributed definitions ...

- A syntax directed definition is L-attributed if each inherited attribute of  $X_j$  ( $1 \leq j \leq n$ ) at the right hand side of  $A \rightarrow X_1 X_2 \dots X_n$  depends only on
  - Attributes of symbols  $X_1 X_2 \dots X_{j-1}$  and
  - Inherited attribute of  $A$
- Examples (i inherited, s synthesized)

$A \rightarrow LM$

$L.i = f_1(A.i)$   
 $M.i = f_2(L.s)$   
 $A.s = f_3(M.s)$



$A \rightarrow QR$

$R.i = f_4(A.i)$   
 $Q.i = f_5(R.s)$   
 $A.s = f_6(Q.s)$

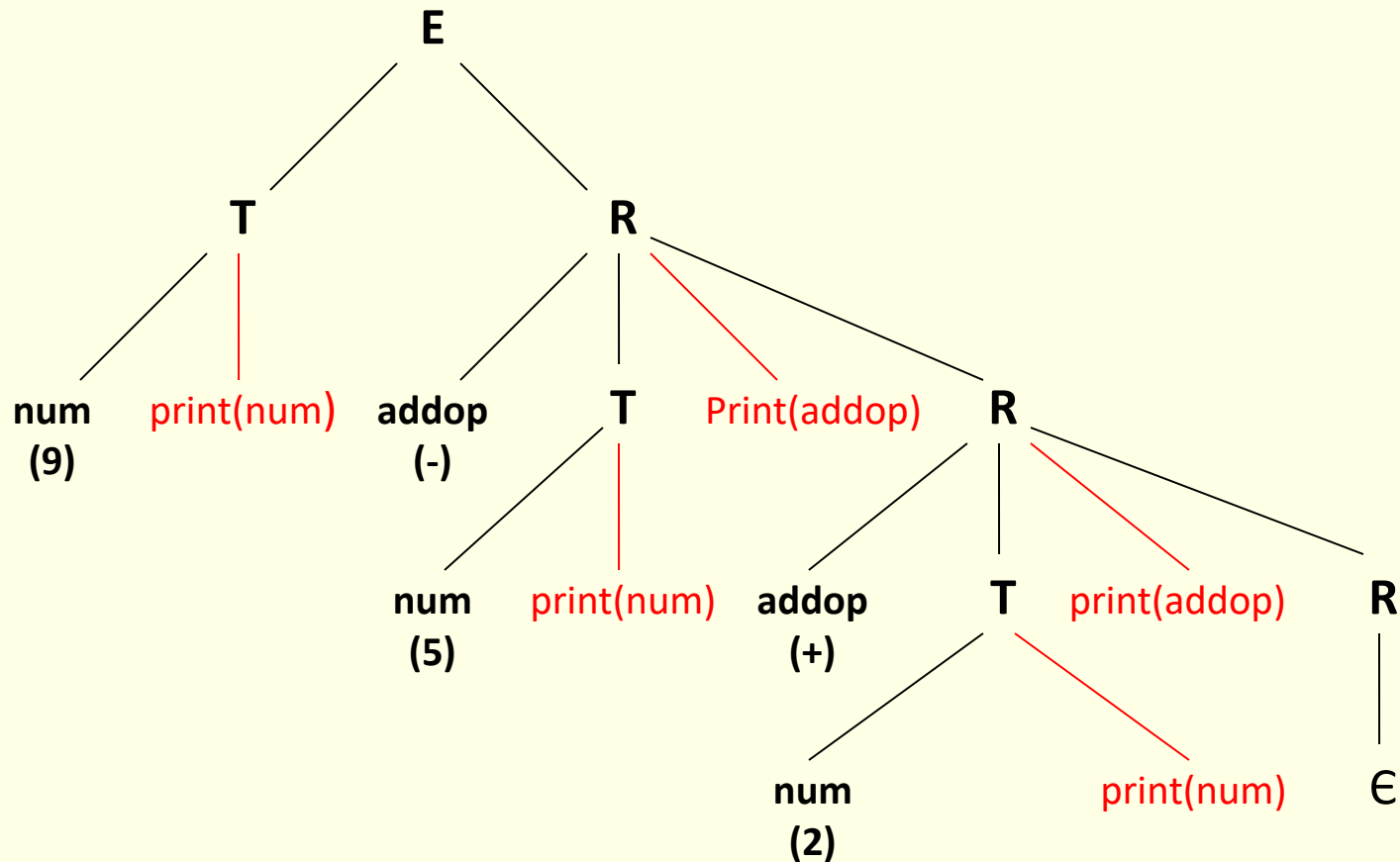


# Translation schemes

- A CFG where semantic actions occur within the rhs of production
- Example: A translation scheme to map infix to postfix  
 $E \rightarrow T R$   
 $R \rightarrow \text{addop } T R \mid \varepsilon$   
 $T \rightarrow \text{num}$   
 $\text{addop} \rightarrow + \mid -$

Exercise: Create Parse Tree for  $9 - 5 + 2$

# Parse tree for 9-5+2



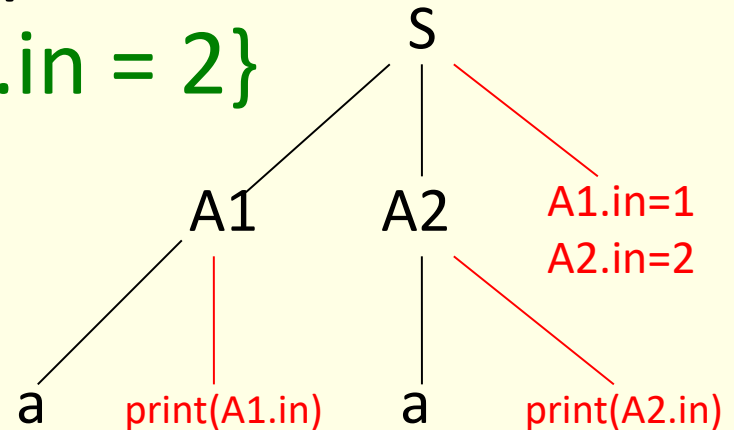


# Evaluation of Translation Schemes

- Assume actions are terminal symbols
- Perform depth first order traversal to obtain  $9\ 5 - 2 +$
- When designing translation scheme, **ensure** attribute value is available when referred to
- In case of synthesized attribute it is trivial (**why ?**)

- An inherited attribute for a symbol on RHS of a production must be computed in an action before that symbol

$S \rightarrow A_1 A_2 \quad \{A_1.in = 1, A_2.in = 2\}$   
 $A \rightarrow a \quad \{\text{print}(A.in)\}$



depth first order traversal gives error (*undef*)

- A synthesized attribute for the non terminal on the LHS can be computed after all attributes it references, have been computed. **The action normally should be placed at the end of RHS.**

# Bottom up evaluation of inherited attributes

- Remove embedded actions from translation scheme
- Make transformation so that embedded actions occur only at the ends of their productions
- Replace each action by a distinct marker non terminal  $M$  and attach action at end of  $M \rightarrow \varepsilon$

$E \rightarrow T R$   
 $R \rightarrow + T \{\text{print (+)}\} R$   
 $R \rightarrow - T \{\text{print (-)}\} R$   
 $R \rightarrow \epsilon$   
 $T \rightarrow \text{num} \{\text{print(num.val)}\}$

transforms to

$E \rightarrow T R$   
 $R \rightarrow + T M R$   
 $R \rightarrow - T N R$   
 $R \rightarrow \epsilon$   
 $T \rightarrow \text{num} \quad \{\text{print(num.val)}\}$   
 $M \rightarrow \epsilon \quad \{\text{print(+)}\}$   
 $N \rightarrow \epsilon \quad \{\text{print(-)}\}$

## Inheriting attribute on parser stacks

- bottom up parser reduces rhs of  $A \rightarrow XY$  by removing  $XY$  from stack and putting  $A$  on the stack
- synthesized attributes of  $Xs$  can be inherited by  $Y$  by using the copy rule  $Y.i = X.s$

# Inherited Attributes: SDD

$D \rightarrow T L$        $L.in = T.type$

$T \rightarrow \text{real}$        $T.type = \text{real}$

$T \rightarrow \text{int}$        $T.type = \text{int}$

$L \rightarrow L_1, \text{id}$        $L_1.in = L.in;$   
                                  $\text{addtype}(\text{id.entry}, L.in)$

$L \rightarrow \text{id}$        $\text{addtype}(\text{id.entry}, L.in)$

Exercise: Convert to Translation Scheme

# Inherited Attributes: Translation Scheme

$D \rightarrow T \{L.in = T.type\} L$

$T \rightarrow \text{int} \quad \{T.type = \text{integer}\}$

$T \rightarrow \text{real} \quad \{T.type = \text{real}\}$

$L \rightarrow \{L_1.in = L.in\} L_1, \text{id} \{addtype(\text{id.entry}, L.in)\}$

$L \rightarrow \text{id} \{addtype(\text{id.entry}, L.in)\}$

**Example:** take string      real p,q,r

State stack	INPUT	PRODUCTION
	real p,q,r	
real	p,q,r	
T	p,q,r	$T \rightarrow \text{real}$
Tp	,q,r	
TL	,q,r	$L \rightarrow \text{id}$
TL,	q,r	
TL,q	,r	
TL	,r	$L \rightarrow L,\text{id}$
TL,	r	
TL,r	-	
TL	-	$L \rightarrow L,\text{id}$
D	-	$D \rightarrow TL$

Every time a string is reduced to L, T.val is just below it on the stack



# Example ...

- Every time a reduction to L is made value of T type is just below it
- Use the fact that T.val (type information) is at a known place in the stack
- When production  $L \rightarrow id$  is applied, id.entry is at the top of the stack and T.type is just below it, therefore,

`addtype(id.entry, L.in)  $\Leftrightarrow$`

`addtype(val[top], val[top-1])`

- Similarly when production  $L \rightarrow L_1$ , id is applied id.entry is at the top of the stack and T.type is three places below it, therefore,

`addtype(id.entry, L.in)  $\Leftrightarrow$`

`addtype(val[top], val[top-3])`

# Example ...

Therefore, the translation scheme becomes

$D \rightarrow T L$

$T \rightarrow \text{int}$        $\text{val}[\text{top}] = \text{integer}$

$T \rightarrow \text{real}$        $\text{val}[\text{top}] = \text{real}$

$L \rightarrow L, \text{id}$        $\text{addtype}(\text{val}[\text{top}], \text{val}[\text{top}-3])$

$L \rightarrow \text{id}$        $\text{addtype}(\text{val}[\text{top}], \text{val}[\text{top}-1])$

# Simulating the evaluation of inherited attributes

- The scheme works only if grammar allows position of attribute to be predicted.
- Consider the grammar
$$\begin{array}{ll} S \rightarrow aAC & C_i = A_s \\ S \rightarrow bABC & C_i = A_s \\ C \rightarrow c & C_s = g(C_i) \end{array}$$
- C inherits  $A_s$
- there may or may not be a B between A and C on the stack when reduction by rule  $C \rightarrow c$  takes place
- When reduction by  $C \rightarrow c$  is performed the value of  $C_i$  is either in [top-1] or [top-2]

# Simulating the evaluation ...

- Insert a marker  $M$  just before  $C$  in the second rule and change rules to

$$S \rightarrow aAC$$

$$S \rightarrow bABMC$$

$$C \rightarrow c$$

$$M \rightarrow \varepsilon$$

$$C_i = A_s$$

$$M_i = A_s; C_i = M_s$$

$$C_s = g(C_i)$$

$$M_s = M_i$$

- When production  $M \rightarrow \varepsilon$  is applied we have  $M_s = M_i = A_s$
- Therefore value of  $C_i$  is always at  $\text{val}[\text{top}-1]$

## Simulating the evaluation ...

- Markers can also be used to simulate rules that are not copy rules

$$S \rightarrow aAC \qquad C_i = f(A.s)$$

- using a marker

$$\begin{array}{l} S \rightarrow aANC \\ N \rightarrow \varepsilon \end{array} \qquad \begin{array}{l} N_i = A_s; C_i = N_s \\ N_s = f(N_i) \end{array}$$

# General algorithm

- **Algorithm:** Bottom up parsing and translation with inherited attributes
- **Input:** L attributed definitions
- **Output:** A bottom up parser
- Assume every non terminal has one inherited attribute and every grammar symbol has a synthesized attribute
- For every production  $A \rightarrow X_1 \dots X_n$  introduce n markers  $M_1 \dots M_n$  and replace the production by
$$\begin{array}{l} A \rightarrow M_1 X_1 \dots M_n X_n \\ M_1 \dots M_n \rightarrow \epsilon \end{array}$$
- Synthesized attribute  $X_{j,s}$  goes into the value entry of  $X_j$
- Inherited attribute  $X_{j,i}$  goes into the value entry of  $M_j$

# Algorithm ...

- If the reduction is to a marker  $M_j$  and the marker belongs to a production

$A \rightarrow M_1 X_1 \dots M_n X_n$  then

$A_j$  is in position  $\text{top}-2j+2$

$X_{1.i}$  is in position  $\text{top}-2j+3$

$X_{1.s}$  is in position  $\text{top}-2j+4$

- If reduction is to a non terminal  $A$  by production  $A \rightarrow M_1 X_1 \dots M_n X_n$  then compute  $A_s$  and push on the stack

# Space for attributes at compile time

- Lifetime of an attribute begins when it is first computed
- Lifetime of an attribute ends when all the attributes depending on it, have been computed
- Space can be conserved by assigning space for an attribute only during its lifetime



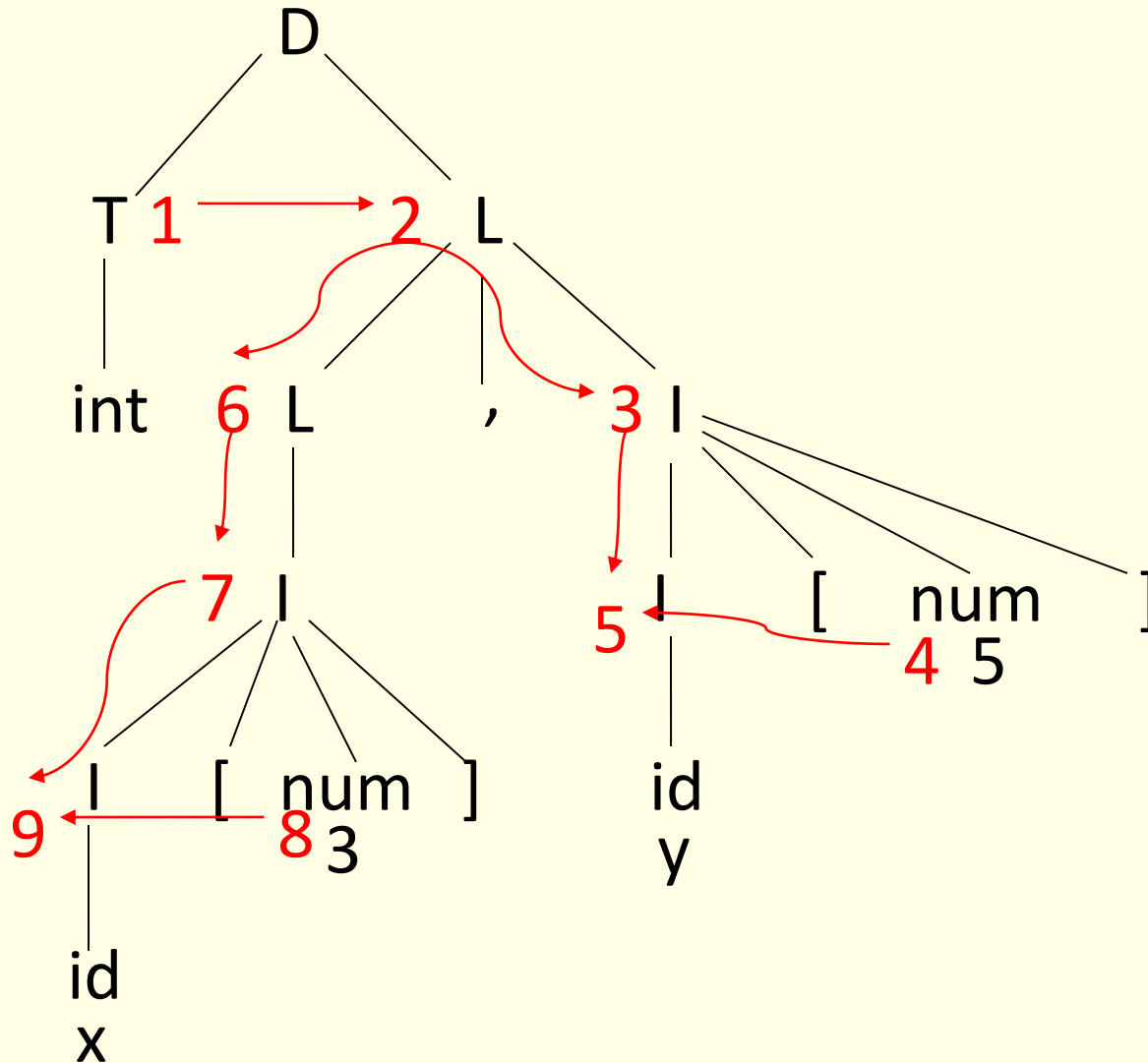
# Example

- Consider following definition

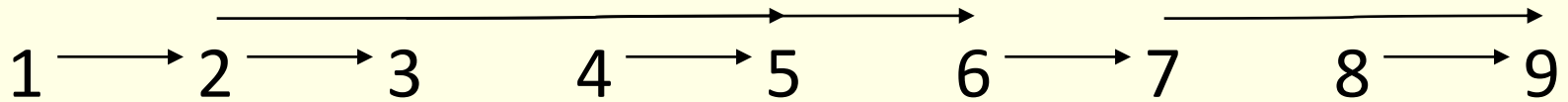
$D \rightarrow T L$	$L.in := T.type$
$T \rightarrow real$	$T.type := real$
$T \rightarrow int$	$T.type := int$
$L \rightarrow L_1, l$	$L_1.in := L.in; l.in = L.in$
$L \rightarrow l$	$l.in = L.in$
$l \rightarrow l_1[num]$	$l_1.in = array(numeral, l.in)$
$l \rightarrow id$	$addtype(id.entry, l.in)$

Consider string `int x[3], y[5]`

its parse tree and dependence graph



# Resource requirement



Allocate resources using life time information

R1 R1 R2 R3 R2 R1 R1 R2 R1

Allocate resources using life time and copy information

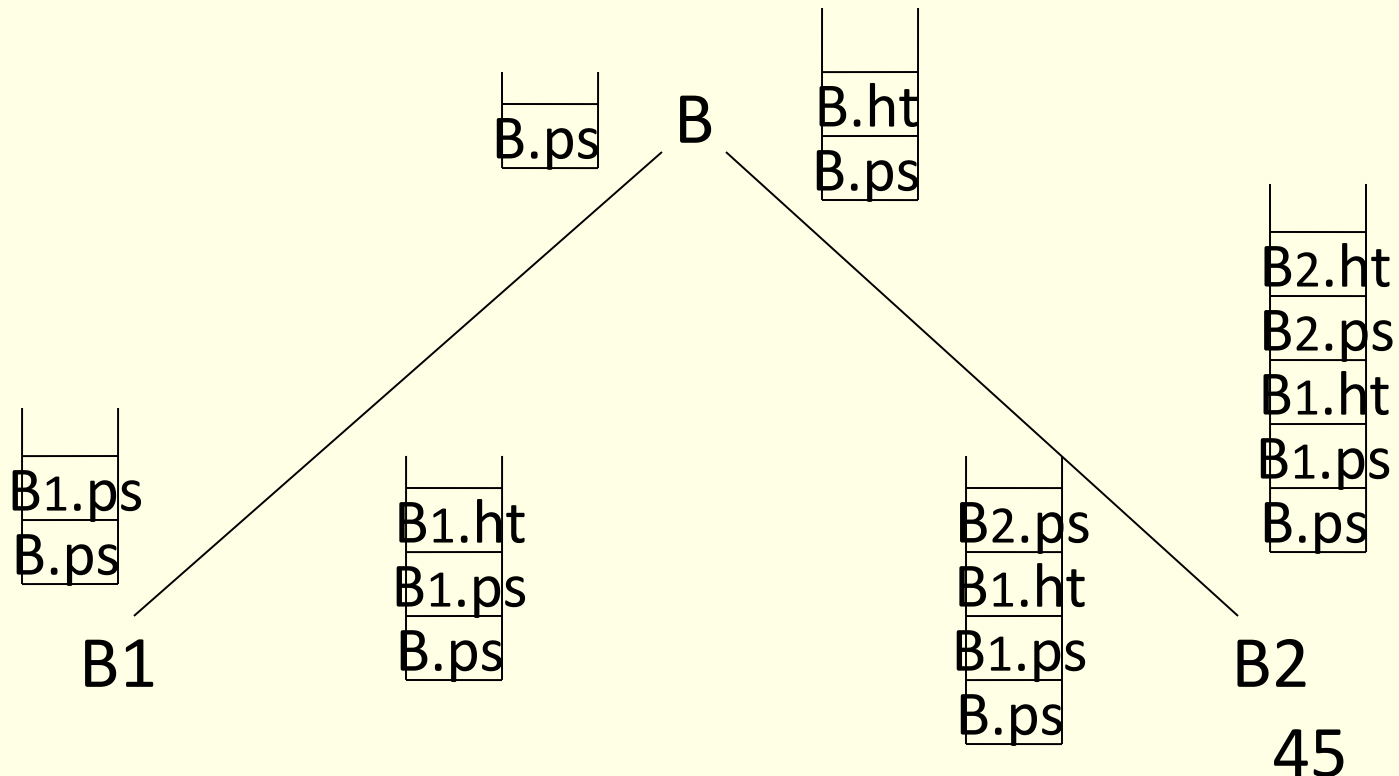
R1 =R1 =R1 R2 R2 =R1 =R1 R2 R1

# Space for attributes at compiler Construction time

- Attributes can be held on a single stack. However, lot of attributes are copies of other attributes
- For a rule like  $A \rightarrow B C$  stack grows up to a height of five (assuming each symbol has one inherited and one synthesized attribute)
- Just before reduction by the rule  $A \rightarrow B C$  the stack contains  $I(A) I(B) S(B) I(C) S(C)$
- After reduction the stack contains  $I(A) S(A)$
-

# Example

- Consider rule  $B \rightarrow B1 B2$  with inherited attribute  $ps$  and synthesized attribute  $ht$
- The parse tree for this string and a snapshot of the stack at each node appears as



# Example ...

- However, if different stacks are maintained for the inherited and synthesized attributes, the stacks will normally be smaller

