

# Bottom up parsing

- Construct a parse tree for an input string beginning at leaves and going towards root OR
- Reduce a string  $w$  of input to start symbol of grammar

Consider a grammar

$$S \rightarrow aABe$$

$$A \rightarrow Abc \mid b$$

$$B \rightarrow d$$

And reduction of a string

$$a \underline{b} b c d e$$

$$a \underline{A} b c d e$$

$$a A \underline{d} e$$

$$\underline{a A B e}$$

$S$

The sentential forms happen to be a *right most derivation in the reverse order*.

$$S \rightarrow a A \underline{B} e$$

$$\rightarrow a \underline{A} d e$$

$$\rightarrow a \underline{A} b c d e$$

$$\rightarrow a b b c d e$$

# Shift reduce parsing

- Split string being parsed into two parts
  - Two parts are separated by a special character “.”
  - Left part is a string of terminals and non terminals
  - Right part is a string of terminals
- Initially the input is `.w`

# Shift reduce parsing ...

- Bottom up parsing has two actions
- **Shift**: move terminal symbol from right string to left string

if string before shift is  $\alpha.pqr$

then string after shift is  $\alpha p.qr$

# Shift reduce parsing ...

- **Reduce**: immediately on the left of “.” identify a string same as RHS of a production and replace it by LHS

if string before reduce action is  $\alpha\beta.pqr$

and  $A \rightarrow \beta$  is a production

then string after reduction is  $\alpha A.pqr$

# Example

Assume grammar is  $E \rightarrow E+E \mid E * E \mid id$

Parse  $id * id + id$

Assume an oracle tells you when to shift / when to reduce

<b>String</b>	<b>action (by oracle)</b>
.id*id+id	shift
id.*id+id	reduce $E \rightarrow id$
E.*id+id	shift
E*.*id+id	shift
E*id.*+id	reduce $E \rightarrow id$
E*E.*+id	reduce $E \rightarrow E * E$
E.*+id	shift
E+.*id	shift
E+id.*	Reduce $E \rightarrow id$
E+E.*	Reduce $E \rightarrow E + E$
E.	<b>ACCEPT</b>

# Shift reduce parsing ...

- Symbols on the left of “.” are kept on a stack
  - Top of the stack is at “.”
  - Shift pushes a terminal on the stack
  - Reduce pops symbols (rhs of production) and pushes a non terminal (lhs of production) onto the stack
- The most important issue: when to shift and when to reduce
- Reduce action should be taken only if the result can be reduced to the start symbol

# Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - Which production to use for reduction?
- Sometimes parser can reduce but it should not:
  - $X \rightarrow \epsilon$  can always be used for reduction!

# Issues in bottom up parsing

- Sometimes parser can reduce in different ways!
- Given stack  $\delta$  and input symbol  $a$ , should the parser
  - Shift  $a$  onto stack (making it  $\delta a$ )
  - Reduce by some production  $A \rightarrow \beta$  assuming that stack has form  $\alpha\beta$  (making it  $\alpha A$ )
  - Stack can have many combinations of  $\alpha\beta$
  - How to keep track of length of  $\beta$ ?



# Handles

- The basic steps of a bottom-up parser are
  - to identify a *substring* within a *rightmost sentential* form which matches the RHS of a rule.
  - when this substring is replaced by the LHS of the matching rule, it must produce the previous rightmost-sentential form.
- Such a substring is called a *handle*

# Handle

- A *handle* of a right sentential form  $\gamma$  is
  - a production rule  $A \rightarrow \beta$ , and
  - an occurrence of a sub-string  $\beta$  in  $\gamma$

such that

- when the occurrence of  $\beta$  is replaced by  $A$  in  $\gamma$ , we get the previous right sentential form in a rightmost derivation of  $\gamma$ .

# Handle

Formally, if

$$S \Rightarrow_{rm^*} \alpha A w \Rightarrow_{rm} \alpha \beta w,$$

then

- $\beta$  in the position following  $\alpha$ ,
  - and the corresponding production  $A \rightarrow \beta$  is a handle of  $\alpha \beta w$ .
- 
- The string  $w$  consists of only terminal symbols

# Handle

- We only want to reduce handle and not any RHS
- **Handle pruning**: If  $\beta$  is a handle and  $A \rightarrow \beta$  is a production then replace  $\beta$  by  $A$
- A right most derivation in reverse can be obtained by handle pruning.

## Handle: Observation

- *Only terminal symbols can appear to the right of a handle in a rightmost sentential form.*
- Why?

# Handle: Observation

Is this scenario possible:

- $\alpha\beta\gamma$  is the content of the stack
- $A \rightarrow \gamma$  is a handle
- The stack content reduces to  $\alpha\beta A$
- Now  $B \rightarrow \beta$  is the handle

In other words, handle is not on top, but buried *inside* stack

Not Possible! Why?

# Handles ...

- Consider two cases of right most derivation to understand the fact that handle appears on the top of the stack

$$S \rightarrow \alpha Az \rightarrow \alpha\beta Byz \rightarrow \alpha\beta\gamma yz$$

$$S \rightarrow \alpha BxAz \rightarrow \alpha Bxyz \rightarrow \alpha\gamma xyz$$

# Handle always appears on the top

Case I:  $S \rightarrow \alpha Az \rightarrow \alpha\beta Byz \rightarrow \alpha\beta\gamma yz$

stack	input	action
$\alpha\beta\gamma$	yz	reduce by $B \rightarrow \gamma$
$\alpha\beta B$	yz	shift y
$\alpha\beta By$	z	reduce by $A \rightarrow \beta By$
$\alpha A$	z	

Case II:  $S \rightarrow \alpha Bx Az \rightarrow \alpha Bxyz \rightarrow \alpha\gamma xyz$

stack	input	action
$\alpha\gamma$	xyz	reduce by $B \rightarrow \gamma$
$\alpha B$	xyz	shift x
$\alpha Bx$	yz	shift y
$\alpha Bxy$	z	reduce $A \rightarrow \gamma$
$\alpha BxA$	z	



# Shift Reduce Parsers

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce
- Bottom up parsing is essentially the process of detecting handles and reducing them.
- Different bottom-up parsers differ in the way they detect handles.

# Conflicts

- What happens when there is a choice
  - What action to take in case both shift and reduce are valid?  
**shift-reduce conflict**
  - Which rule to use for reduction if reduction is possible by more than one rule?  
**reduce-reduce conflict**

# Conflicts

- Conflicts come either because of ambiguous grammars or parsing method is not powerful enough

# Shift reduce conflict

Consider the grammar  $E \rightarrow E+E \mid E^*E \mid id$

and the input  $id+id^*id$

stack	input	action
E+E	*id	reduce by $E \rightarrow E+E$
E	*id	shift
E*	id	shift
E*id		reduce by $E \rightarrow id$
E*E		reduce by $E \rightarrow E^*E$
E		

stack	input	action
E+E	*id	shift
E+E*	id	shift
E+E*id		reduce by $E \rightarrow id$
E+E*E		reduce by $E \rightarrow E^*E$
E+E		reduce by $E \rightarrow E+E$
E		

# Reduce reduce conflict

Consider the grammar  $M \rightarrow R+R \mid R+c \mid R$

$R \rightarrow c$

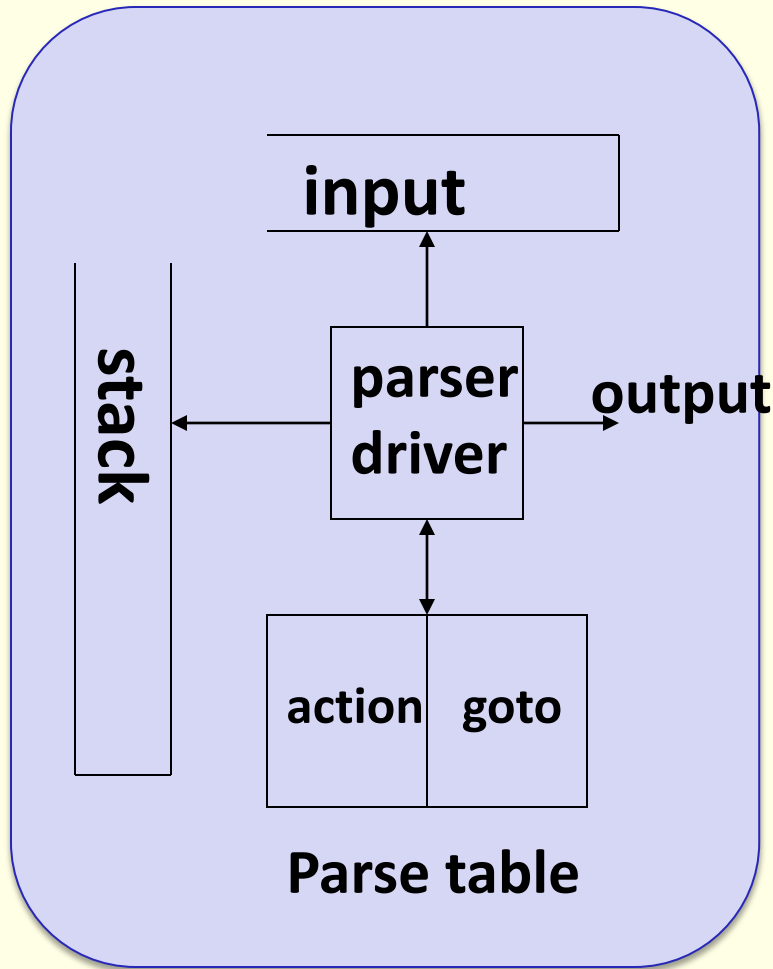
and the input

$c+c$

Stack	input	action
	c+c	shift
c	+c	reduce by $R \rightarrow c$
R	+c	shift
R+	c	shift
R+c		reduce by $R \rightarrow c$
R+R		reduce by $M \rightarrow R+R$
M		

Stack	input	action
	c+c	shift
c	+c	reduce by $R \rightarrow c$
R	+c	shift
R+	c	shift
R+c		reduce by $M \rightarrow R+c$
M		

# LR parsing



- Input buffer contains the input string.
- Stack contains a string of the form  $S_0X_1S_1X_2\dots X_nS_n$  where each  $X_i$  is a grammar symbol and each  $S_i$  is a state.
- Table contains action and goto parts.
- action table is indexed by state and terminal symbols.
- goto table is indexed by state and non terminal symbols.

# Example

Consider a grammar  
and its parse table

$$\begin{array}{l} E \rightarrow E + T \mid T \\ T \rightarrow T * F \mid F \\ F \rightarrow ( E ) \mid \text{id} \end{array}$$

State	id	+	*	(	)	\$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

action

goto

# Actions in an LR (shift reduce) parser

- Assume  $S_i$  is top of stack and  $a_i$  is current input symbol
- Action  $[S_i, a_i]$  can have four values
  1.  $sj$ : shift  $a_i$  to the stack, goto state  $S_j$
  2.  $rk$ : reduce by rule number  $k$
  3.  $acc$ : Accept
  4.  $err$ : Error (empty cells in the table)



# Driving the LR parser

**Stack:**  $S_0 X_1 S_1 X_2 \dots X_m S_m$       **Input:**  $a_i a_{i+1} \dots a_n \$$

- If  $\text{action}[S_m, a_i] = \text{shift } S$

Then the configuration becomes

**Stack:**  $S_0 X_1 S_1 \dots X_m S_m a_i S$       **Input:**  $a_{i+1} \dots a_n \$$

- If  $\text{action}[S_m, a_i] = \text{reduce } A \rightarrow \beta$

Then the configuration becomes

**Stack:**  $S_0 X_1 S_1 \dots X_{m-r} S_{m-r} A S$       **Input:**  $a_i a_{i+1} \dots a_n \$$

Where  $r = |\beta|$  and  $S = \text{goto}[S_{m-r}, A]$

# Driving the LR parser

Stack:  $S_0 X_1 S_1 X_2 \dots X_m S_m$     Input:  $a_i a_{i+1} \dots a_n \$$

- If  $\text{action}[S_m, a_i] = \text{accept}$   
Then parsing is completed. HALT
- If  $\text{action}[S_m, a_i] = \text{error}$  (or empty cell)  
Then invoke error recovery routine.

# Parse $id + id * id$

Stack	Input	Action
0	$id+id*id\$$	shift 5
0 id 5	$+id*id\$$	reduce by $F \rightarrow id$
0 F 3	$+id*id\$$	reduce by $T \rightarrow F$
0 T 2	$+id*id\$$	reduce by $E \rightarrow T$
0 E 1	$+id*id\$$	shift 6
0 E 1 + 6	$id*id\$$	shift 5
0 E 1 + 6 id 5	$*id\$$	reduce by $F \rightarrow id$
0 E 1 + 6 F 3	$*id\$$	reduce by $T \rightarrow F$
0 E 1 + 6 T 9	$*id\$$	shift 7
0 E 1 + 6 T 9 * 7	$id\$$	shift 5
0 E 1 + 6 T 9 * 7 id 5	$\$$	reduce by $F \rightarrow id$
0 E 1 + 6 T 9 * 7 F 10	$\$$	reduce by $T \rightarrow T * F$
0 E 1 + 6 T 9	$\$$	reduce by $E \rightarrow E + T$
0 E 1	$\$$	<b>ACCEPT</b>

# Configuration of a LR parser

- The tuple

$\langle \text{Stack Contents, Remaining Input} \rangle$   
defines a *configuration* of a LR parser

- Initially the configuration is

$$\langle S_0, a_0 a_1 \dots a_n \$ \rangle$$

- Typical final configuration on a successful parse is

$$\langle S_0 X_1 S_i, \$ \rangle$$

# LR parsing Algorithm

Initial state:            **Stack:**  $S_0$     **Input:**  $w\$$

```
while (1) {  
  if (action[S,a] = shift  $S'$ ) {  
    push(a); push( $S'$ ); ip++  
  } else if (action[S,a] = reduce  $A \rightarrow \beta$ ) {  
    pop ( $2 * |\beta|$ ) symbols;  
    push(A); push (goto[ $S''$ ,A])  
    ( $S''$  is the state at stack top after popping symbols)  
  } else if (action[S,a] = accept) {  
    exit  
  } else { error }  
}
```

# Constructing parse table

## Augment the grammar

- $G$  is a grammar with start symbol  $S$
- The augmented grammar  $G'$  for  $G$  has a new start symbol  $S'$  and an additional production  $S' \rightarrow S$
- When the parser reduces by this rule it will stop with accept

# Production to Use for Reduction

- How do we know which production to apply in a given configuration
- We can guess!
  - May require backtracking
- Keep track of “ALL” possible rules that can apply at a given point in the input string
  - But in general, there is no upper bound on the length of the input string
  - Is there a bound on number of applicable rules?

# Some hands on!

- $E' \rightarrow E$
- $E \rightarrow E + T$
- $E \rightarrow T$
- $T \rightarrow T * F$
- $T \rightarrow F$
- $F \rightarrow (E)$
- $F \rightarrow id$

## Strings to Parse

- $id + id + id + id$
- $id * id * id * id$
- $id * id + id * id$
- $id * (id + id) * id$



# Parser states

- Goal is to know the valid reductions at any given point
- Summarize all possible stack prefixes  $\alpha$  as a parser state
- Parser state is defined by a DFA state that reads in the stack  $\alpha$
- Accept states of DFA are unique reductions

# Viable prefixes

- $\alpha$  is a viable prefix of the grammar if
  - $\exists w$  such that  $\alpha w$  is a right sentential form
  - $\langle \alpha, w \rangle$  is a configuration of the parser
- As long as the parser has viable prefixes on the stack no parser error has been seen
- The set of viable prefixes is a regular language
- We can construct an automaton that accepts viable prefixes

# LR(0) items

- An LR(0) item of a grammar  $G$  is a production of  $G$  with a special symbol “.” at some position of the right side
- Thus production  $A \rightarrow XYZ$  gives four LR(0) items

$A \rightarrow .XYZ$

$A \rightarrow X.YZ$

$A \rightarrow XY.Z$

$A \rightarrow XYZ.$

# LR(0) items

- An item indicates how much of a production has been seen at a point in the process of parsing
  - Symbols on the left of “.” are already on the stacks
  - Symbols on the right of “.” are expected in the input

# Start state

- Start state of DFA is an empty stack corresponding to  $S' \rightarrow .S$  item
- This means no input has been seen
- The parser expects to see a string derived from  $S$

# Closure of a state

- **Closure** of a state adds items for all productions whose LHS occurs in an item in the state, just after “.”
  - Set of possible productions to be reduced next
  - Added items have “.” located at the beginning
  - No symbol of these items is on the stack as yet

# Closure operation

- Let  $I$  be a set of items for a grammar  $G$
- $\text{closure}(I)$  is a set constructed as follows:
  - Every item in  $I$  is in  $\text{closure}(I)$
  - If  $A \rightarrow \alpha.B\beta$  is in  $\text{closure}(I)$  and  $B \rightarrow \gamma$  is a production then  $B \rightarrow .\gamma$  is in  $\text{closure}(I)$
- Intuitively  $A \rightarrow \alpha.B\beta$  indicates that we expect a string derivable from  $B\beta$  in input
- If  $B \rightarrow \gamma$  is a production then we might see a string derivable from  $\gamma$  at this point

# Example

For the grammar

$$E' \rightarrow E$$
$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow ( E ) \mid \text{id}$$

If  $I$  is  $\{ E' \rightarrow .E \}$  then  
 $\text{closure}(I)$  is

$$E' \rightarrow .E$$
$$E \rightarrow .E + T$$
$$E \rightarrow .T$$
$$T \rightarrow .T * F$$
$$T \rightarrow .F$$
$$F \rightarrow .\text{id}$$
$$F \rightarrow .(E)$$



# Goto operation

- $\text{Goto}(I, X)$  , where  $I$  is a set of items and  $X$  is a grammar symbol,
  - is closure of set of item  $A \rightarrow \alpha X \beta$
  - such that  $A \rightarrow \alpha X \beta$  is in  $I$
- Intuitively if  $I$  is a set of items for some valid prefix  $\alpha$  then  $\text{goto}(I, X)$  is set of valid items for prefix  $\alpha X$

# Goto operation

If  $I$  is  $\{ E' \rightarrow E. , E \rightarrow E. + T \}$  then  
 $\text{goto}(I, +)$  is

$E \rightarrow E + .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

# Sets of items

C : Collection of sets of LR(0) items for  
grammar  $G'$

$C = \{ \text{closure} ( \{ S' \rightarrow .S \} ) \}$

repeat

    for each set of items  $I$  in  $C$

        for each grammar symbol  $X$

            if  $\text{goto}(I, X)$  is not empty and not in  $C$

                ADD  $\text{goto}(I, X)$  to  $C$

until no more additions to  $C$

# Example

Grammar:

$E' \rightarrow E$

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

$I_0$ : closure( $E' \rightarrow .E$ )

$E' \rightarrow .E$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

$I_1$ : goto( $I_0, E$ )

$E' \rightarrow E.$

$E \rightarrow E. + T$

$I_2$ : goto( $I_0, T$ )

$E \rightarrow T.$

$T \rightarrow T. * F$

$I_3$ : goto( $I_0, F$ )

$T \rightarrow F.$

$I_4$ : goto( $I_0, ($ )

$F \rightarrow (.E)$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

$I_5$ : goto( $I_0, id$ )

$F \rightarrow id.$

$I_6: \text{goto}(I_1, +)$   
 $E \rightarrow E + \cdot T$   
 $T \rightarrow \cdot T * F$   
 $T \rightarrow \cdot F$   
 $F \rightarrow \cdot (E)$   
 $F \rightarrow \cdot \text{id}$

$I_9: \text{goto}(I_6, T)$   
 $E \rightarrow E + T \cdot$   
 $T \rightarrow T \cdot * F$

$\text{goto}(I_6, F)$  is  $I_3$   
 $\text{goto}(I_6, ( ))$  is  $I_4$   
 $\text{goto}(I_6, \text{id})$  is  $I_5$

$I_7: \text{goto}(I_2, *)$   
 $T \rightarrow T * \cdot F$   
 $F \rightarrow \cdot (E)$   
 $F \rightarrow \cdot \text{id}$

$I_{10}: \text{goto}(I_7, F)$   
 $T \rightarrow T * F \cdot$

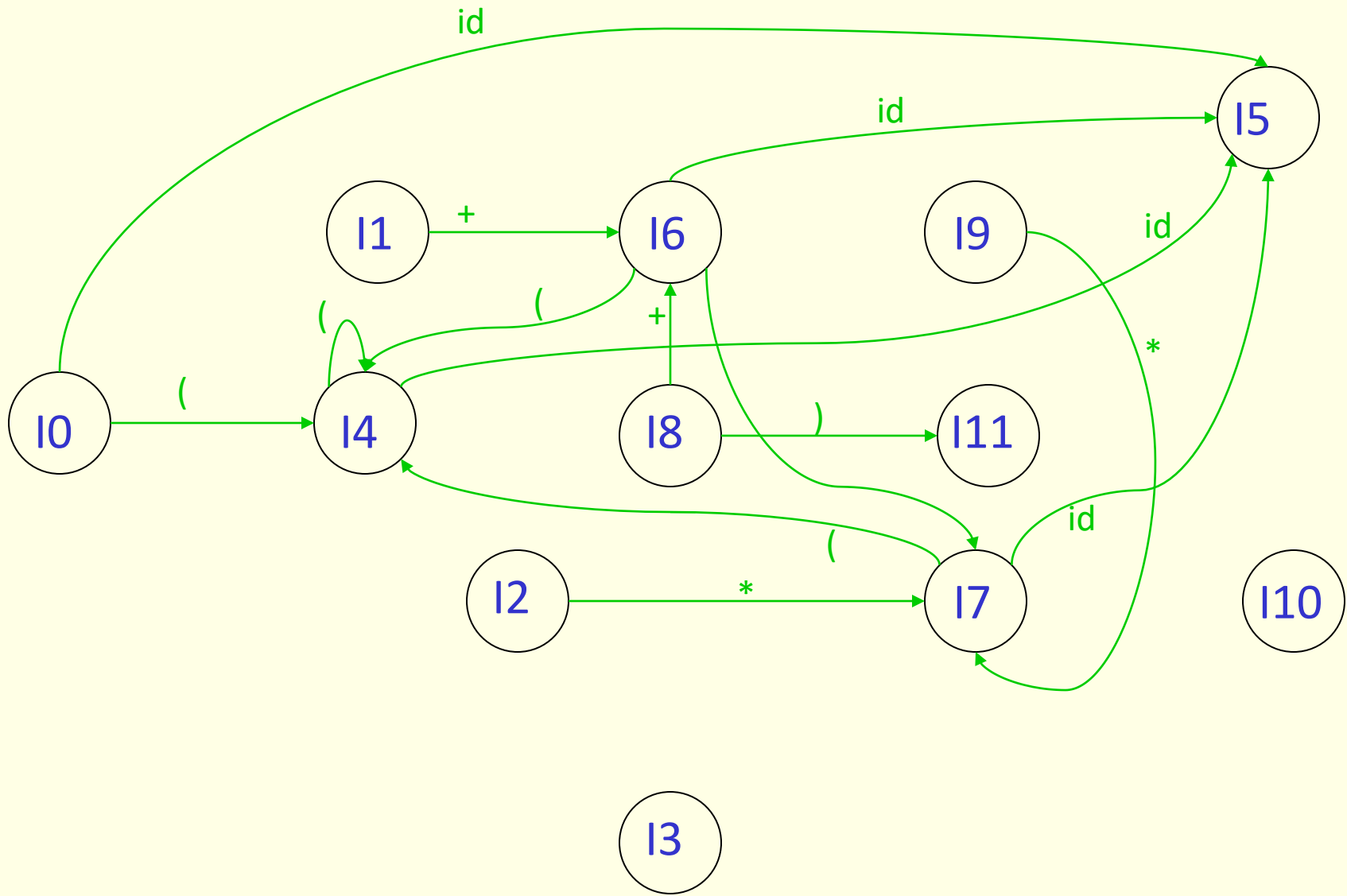
$\text{goto}(I_7, ( ))$  is  $I_4$   
 $\text{goto}(I_7, \text{id})$  is  $I_5$

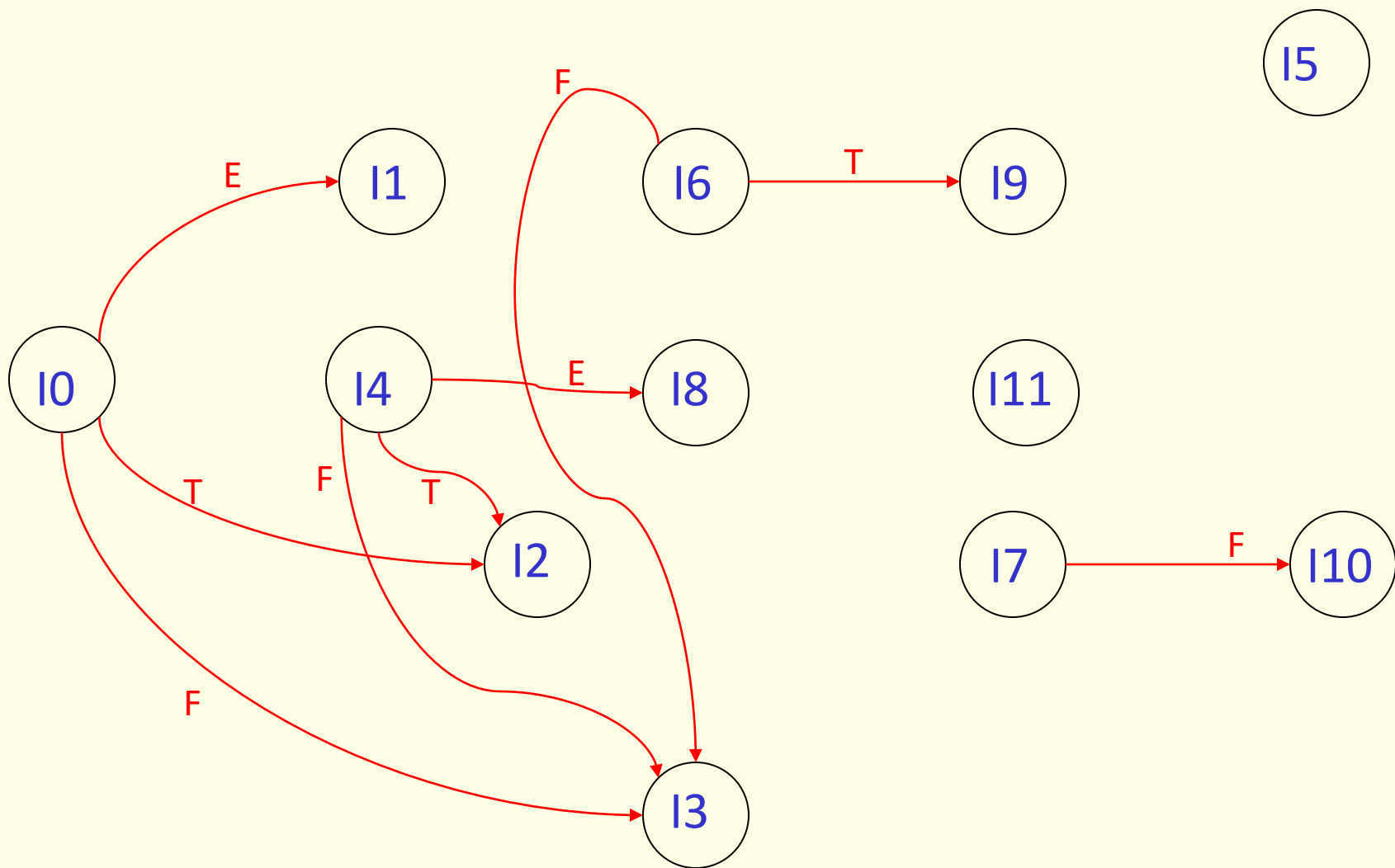
$I_8: \text{goto}(I_4, E)$   
 $F \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot + T$

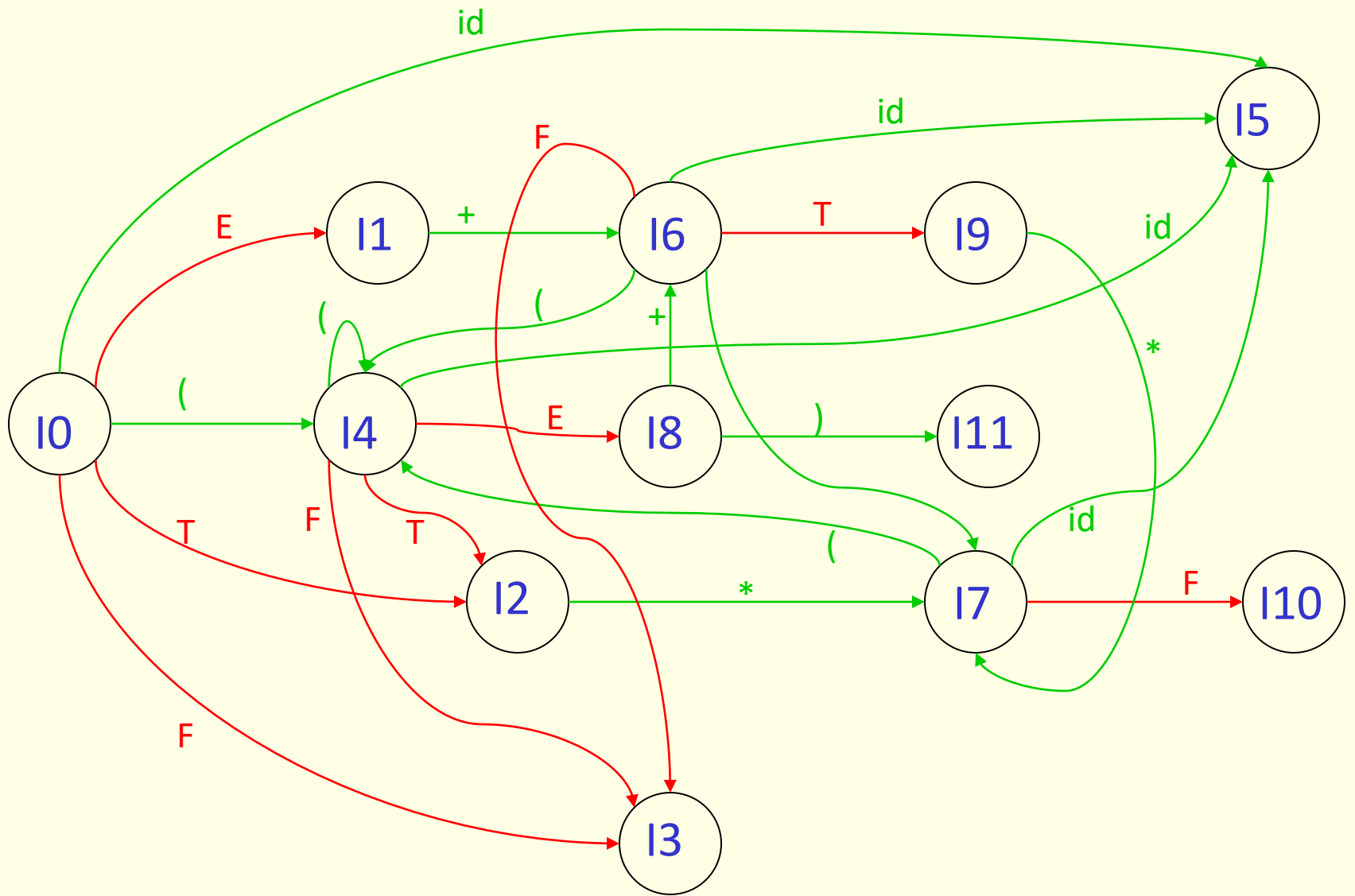
$I_{11}: \text{goto}(I_8, )$   
 $F \rightarrow (E) \cdot$

$\text{goto}(I_4, T)$  is  $I_2$   
 $\text{goto}(I_4, F)$  is  $I_3$   
 $\text{goto}(I_4, ( ))$  is  $I_4$   
 $\text{goto}(I_4, \text{id})$  is  $I_5$

$\text{goto}(I_8, +)$  is  $I_6$   
 $\text{goto}(I_9, *)$  is  $I_7$









# LR(0) (?) Parse Table

- The information is still not sufficient to help us resolve shift-reduce conflict.

For example the state:

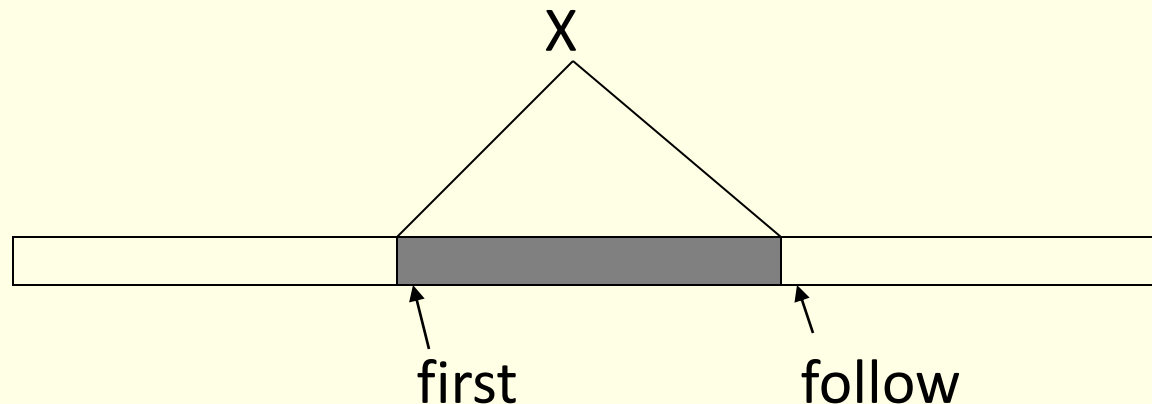
$$I_1: E' \rightarrow E.$$

$$E \rightarrow E. + T$$

- We need some more information to make decisions.

# Constructing parse table

- **First( $\alpha$ )** for a string of terminals and non terminals  $\alpha$  is
  - Set of symbols that might begin the fully expanded (made of only tokens) version of  $\alpha$
- **Follow( $X$ )** for a non terminal  $X$  is
  - set of symbols that might follow the derivation of  $X$  in the input stream



# Compute first sets

- If  $X$  is a terminal symbol then  $\text{first}(X) = \{X\}$
- If  $X \rightarrow \epsilon$  is a production then  $\epsilon$  is in  $\text{first}(X)$
- If  $X$  is a non terminal and  $X \rightarrow Y_1 Y_2 \dots Y_k$  is a production, then
  - if for some  $i$ ,  $a$  is in  $\text{first}(Y_i)$
  - and  $\epsilon$  is in all of  $\text{first}(Y_j)$  (such that  $j < i$ )
  - then  $a$  is in  $\text{first}(X)$
- If  $\epsilon$  is in  $\text{first}(Y_1) \dots \text{first}(Y_k)$  then  $\epsilon$  is in  $\text{first}(X)$
- Now generalize to a string  $\alpha$  of terminals and non-terminals

# Example

- For the expression grammar

$$E \rightarrow T E' \qquad E' \rightarrow +T E' \mid \epsilon$$

$$T \rightarrow F T' \qquad T' \rightarrow * F T' \mid \epsilon$$

$$F \rightarrow ( E ) \mid \text{id}$$

$$\begin{aligned} \text{First}(E) &= \text{First}(T) = \text{First}(F) \\ &= \{ (, \text{id} \} \end{aligned}$$

$$\begin{aligned} \text{First}(E') \\ &= \{ +, \epsilon \} \end{aligned}$$

$$\begin{aligned} \text{First}(T') \\ &= \{ *, \epsilon \} \end{aligned}$$

# Compute follow sets

1. Place  $\$$  in  $\text{follow}(S)$  //  $S$  is the start symbol
  2. If there is a production  $A \rightarrow \alpha B \beta$   
then everything in  $\text{first}(\beta)$  (except  $\epsilon$ ) is in  $\text{follow}(B)$
  3. If there is a production  $A \rightarrow \alpha B \beta$  and  $\text{first}(\beta)$  contains  $\epsilon$   
then everything in  $\text{follow}(A)$  is in  $\text{follow}(B)$
  4. If there is a production  $A \rightarrow \alpha B$   
then everything in  $\text{follow}(A)$  is in  $\text{follow}(B)$
- Last two steps have to be repeated until the follow sets converge.

# Example

- For the expression grammar

$$E \rightarrow T E'$$

$$E' \rightarrow + T E' \mid \epsilon$$

$$T \rightarrow F T'$$

$$T' \rightarrow * F T' \mid \epsilon$$

$$F \rightarrow ( E ) \mid \text{id}$$

$$\text{follow}(E) = \text{follow}(E') = \{ \$, ) \}$$

$$\text{follow}(T) = \text{follow}(T') = \{ \$, ), + \}$$

$$\text{follow}(F) = \{ \$, ), +, * \}$$

# Construct SLR parse table

- Construct  $C = \{I_0, \dots, I_n\}$  the collection of sets of LR(0) items
- If  $A \rightarrow \alpha.a\beta$  is in  $I_i$  and  $\text{goto}(I_i, a) = I_j$  then  $\text{action}[i, a] = \text{shift } j$
- If  $A \rightarrow \alpha.$  is in  $I_i$  then  $\text{action}[i, a] = \text{reduce } A \rightarrow \alpha$  for all  $a$  in  $\text{follow}(A)$
- If  $S' \rightarrow S.$  is in  $I_i$  then  $\text{action}[i, \$] = \text{accept}$
- If  $\text{goto}(I_i, A) = I_j$  then  $\text{goto}[i, A] = j$  for all non terminals  $A$
- All entries not defined are errors

# Notes

- This method of parsing is called SLR (Simple LR)
- LR parsers accept LR(k) languages
  - L stands for left to right scan of input
  - R stands for rightmost derivation
  - k stands for number of lookahead token
- SLR is the simplest of the LR parsing methods. SLR is too weak to handle most languages!
- If an SLR parse table for a grammar does not have multiple entries in any cell then the grammar is unambiguous
- All SLR grammars are unambiguous
- Are all unambiguous grammars in SLR?



# Practice Assignment

Construct SLR parse table for following grammar

$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid ( E ) \mid \text{digit}$$

Show steps in parsing of string

$$9*5+(2+3*7)$$

- Steps to be followed
  - Augment the grammar
  - Construct set of LR(0) items
  - Construct the parse table
  - Show states of parser as the given string is parsed

# Example

- Consider following grammar and its SLR parse table:

$S' \rightarrow S$

$S \rightarrow L = R$

$S \rightarrow R$

$L \rightarrow *R$

$L \rightarrow id$

$R \rightarrow L$

$I_1: \text{goto}(I_0, S)$

$S' \rightarrow S.$

$I_2: \text{goto}(I_0, L)$

$S \rightarrow L.=R$

$R \rightarrow L.$

$I_0: S' \rightarrow .S$

$S \rightarrow .L=R$

$S \rightarrow .R$

$L \rightarrow .*R$

$L \rightarrow .id$

$R \rightarrow .L$

**Assignment** (not to be submitted):  
Construct rest of the items and the parse table.

## SLR parse table for the grammar

	=	*	id	\$	S	L	R
0		s4	s5		1	2	3
1				acc			
2	s6,r6			r6			
3				r3			
4		s4	s5			8	7
5	r5			r5			
6		s4	s5			8	9
7	r4			r4			
8	r6			r6			
9				r2			

The table has multiple entries in action[2,=]

- There is both a shift and a reduce entry in action[2,=]. Therefore state 2 has a shift-reduce conflict on symbol "=", However, the grammar is not ambiguous.
- Parse id=id assuming reduce action is taken in [2,=]

<b>Stack</b>	<b>input</b>	<b>action</b>
0	id=id	shift 5
0 id 5	=id	reduce by $L \rightarrow id$
0 L 2	=id	reduce by $R \rightarrow L$
0 R 3	=id	<b>error</b>

- if shift action is taken in [2,=]

<b>Stack</b>	<b>input</b>	<b>action</b>
0	id=id\$	shift 5
0 id 5	=id\$	reduce by L→id
0 L 2	=id\$	shift 6
0 L 2 = 6	id\$	shift 5
0 L 2 = 6 id 5	\$	reduce by L→id
0 L 2 = 6 L 8	\$	reduce by R→L
0 L 2 = 6 R 9	\$	reduce by S→L=R
0 S 1	\$	<b>ACCEPT</b>

# Problems in SLR parsing

- No sentential form of this grammar can start with  $R=...$
- However, the reduce action in action[2,=] generates a sentential form starting with  $R=$
- Therefore, the reduce action is incorrect
- In SLR parsing method state  $i$  calls for reduction on symbol “a”, by rule  $A \rightarrow \alpha$  if  $I_i$  contains  $[A \rightarrow \alpha.]$  and “a” is in  $\text{follow}(A)$
- However, when state  $I$  appears on the top of the stack, the viable prefix  $\beta\alpha$  on the stack may be such that  $\beta A$  can not be followed by symbol “a” in any right sentential form
- Thus, the reduction by the rule  $A \rightarrow \alpha$  on symbol “a” is invalid
- **SLR parsers cannot remember the left context**

# Canonical LR Parsing

- Carry extra information in the state so that wrong reductions by  $A \rightarrow \alpha$  will be ruled out
- Redefine LR items to include a terminal symbol as a second component (look ahead symbol)
- The general form of the item becomes  $[A \rightarrow \alpha.\beta, a]$  which is called LR(1) item.
- Item  $[A \rightarrow \alpha., a]$  calls for reduction only if next input is  $a$ . The set of symbols “ $a$ ”s will be a subset of  $\text{Follow}(A)$ .

# Closure(I)

repeat

  for each item  $[A \rightarrow \alpha.B\beta, a]$  in I

    for each production  $B \rightarrow \gamma$  in  $G'$

    and for each terminal  $b$  in  $\text{First}(\beta a)$

      add item  $[B \rightarrow .\gamma, b]$  to I

until no more additions to I



# Example

Consider the following grammar

$$S' \rightarrow S$$

$$S \rightarrow CC$$

$$C \rightarrow cC \mid d$$

Compute  $\text{closure}(I)$  where  $I = \{[S' \rightarrow .S, \$]\}$

$S' \rightarrow .S,$	$\$$
$S \rightarrow .CC,$	$\$$
$C \rightarrow .cC,$	$c$
$C \rightarrow .cC,$	$d$
$C \rightarrow .d,$	$c$
$C \rightarrow .d,$	$d$

# Example

Construct sets of LR(1) items for the grammar on previous slide

$l_0: S' \rightarrow .S,$	\$	$l_4: \text{goto}(l_0, d)$	
$S \rightarrow .CC,$	\$	$C \rightarrow d.,$	c/d
$C \rightarrow .cC,$	c/d		
$C \rightarrow .d,$	c/d	$l_5: \text{goto}(l_2, C)$	
		$S \rightarrow \dot{C}C.,$	\$
$l_1: \text{goto}(l_0, S)$			
$S' \rightarrow S.,$	\$	$l_6: \text{goto}(l_2, c)$	
		$C \rightarrow c.C,$	\$
$l_2: \text{goto}(l_0, C)$		$C \rightarrow .cC,$	\$
$S \rightarrow C.C,$	\$	$C \rightarrow .d,$	\$
$C \rightarrow .cC,$	\$		
$C \rightarrow .d,$	\$	$l_7: \text{goto}(l_2, d)$	
		$C \rightarrow d.,$	\$
$l_3: \text{goto}(l_0, c)$			
$C \rightarrow c.C,$	c/d	$l_8: \text{goto}(l_3, C)$	
$C \rightarrow .cC,$	c/d	$C \rightarrow c\dot{C}.,$	c/d
$C \rightarrow .d,$	c/d		
		$l_9: \text{goto}(l_6, C)$	
		$C \rightarrow c\dot{C}.,$	\$

# Construction of Canonical LR parse table

- Construct  $C = \{I_0, \dots, I_n\}$  the sets of LR(1) items.
- If  $[A \rightarrow \alpha.a\beta, b]$  is in  $I_i$  and  $\text{goto}(I_i, a) = I_j$   
then  $\text{action}[i, a] = \text{shift } j$
- If  $[A \rightarrow \alpha., a]$  is in  $I_i$   
then  $\text{action}[i, a] = \text{reduce } A \rightarrow \alpha$
- If  $[S' \rightarrow S., \$]$  is in  $I_i$   
then  $\text{action}[i, \$] = \text{accept}$
- If  $\text{goto}(I_i, A) = I_j$  then  $\text{goto}[i, A] = j$  for all non terminals  $A$

# Parse table

State	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

# Notes on Canonical LR Parser

- Consider the grammar discussed in the previous two slides. The language specified by the grammar is  $c^*dc^*d$ .
- When reading input  $cc\dots dcc\dots d$  the parser shifts  $cs$  into stack and then goes into state 4 after reading  $d$ . It then calls for reduction by  $C \rightarrow d$  if following symbol is  $c$  or  $d$ .
- IF  $\$$  follows the first  $d$  then input string is  $c^*d$  which is not in the language; parser declares an error
- On an error canonical LR parser never makes a wrong shift/reduce move. It immediately declares an error
- **Problem:** Canonical LR parse table has a large number of states

# LALR Parse table

- Look Ahead LR parsers
- Consider a pair of similar looking states (same kernel and different lookaheads) in the set of LR(1) items  
 $I_4: C \rightarrow d. , c/d$                        $I_7: C \rightarrow d., \$$
- Replace  $I_4$  and  $I_7$  by a new state  $I_{47}$  consisting of  $(C \rightarrow d., c/d/\$)$
- Similarly  $I_3$  &  $I_6$  and  $I_8$  &  $I_9$  form pairs
- Merge LR(1) items having the same core

# Construct LALR parse table

- Construct  $C = \{I_0, \dots, I_n\}$  set of LR(1) items
- For each core present in LR(1) items find all sets having the same core and replace these sets by their union
- Let  $C' = \{J_0, \dots, J_m\}$  be the resulting set of items
- Construct action table as was done earlier
- Let  $J = I_1 \cup I_2 \dots \cup I_k$

since  $I_1, I_2, \dots, I_k$  have same core,  $\text{goto}(J, X)$  will have the same core

Let  $K = \text{goto}(I_1, X) \cup \text{goto}(I_2, X) \dots \cup \text{goto}(I_k, X)$  then  $\text{goto}(J, X) = K$

# LALR parse table ...

State	c	d	\$	S	C
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		



# Notes on LALR parse table

- Modified parser behaves as original except that it will reduce  $C \rightarrow d$  on inputs like  $ccd$ . The error will eventually be caught before any more symbols are shifted.
- In general core is a set of LR(0) items and LR(1) grammar may produce more than one set of items with the same core.
- Merging items never produces shift/reduce conflicts but may produce reduce/reduce conflicts.
- SLR and LALR parse tables have same number of states.

# Notes on LALR parse table...

- Merging items may result into conflicts in LALR parsers which did not exist in LR parsers
- New conflicts can not be of shift reduce kind:
  - Assume there is a shift reduce conflict in some state of LALR parser with items  $\{[X \rightarrow \alpha., a], [Y \rightarrow \gamma.a\beta, b]\}$
  - Then there must have been a state in the LR parser with the same core
  - Contradiction; because LR parser did not have conflicts
- LALR parser can have new reduce-reduce conflicts
  - Assume states  $\{[X \rightarrow \alpha., a], [Y \rightarrow \beta., b]\}$  and  $\{[X \rightarrow \alpha., b], [Y \rightarrow \beta., a]\}$
  - Merging the two states produces  $\{[X \rightarrow \alpha., a/b], [Y \rightarrow \beta., a/b]\}$

# Notes on LALR parse table...

- LALR parsers are not built by first making canonical LR parse tables
- There are direct, complicated but efficient algorithms to develop LALR parsers
- Relative power of various classes
  - $SLR(1) \leq LALR(1) \leq LR(1)$
  - $SLR(k) \leq LALR(k) \leq LR(k)$
  - $LL(k) \leq LR(k)$

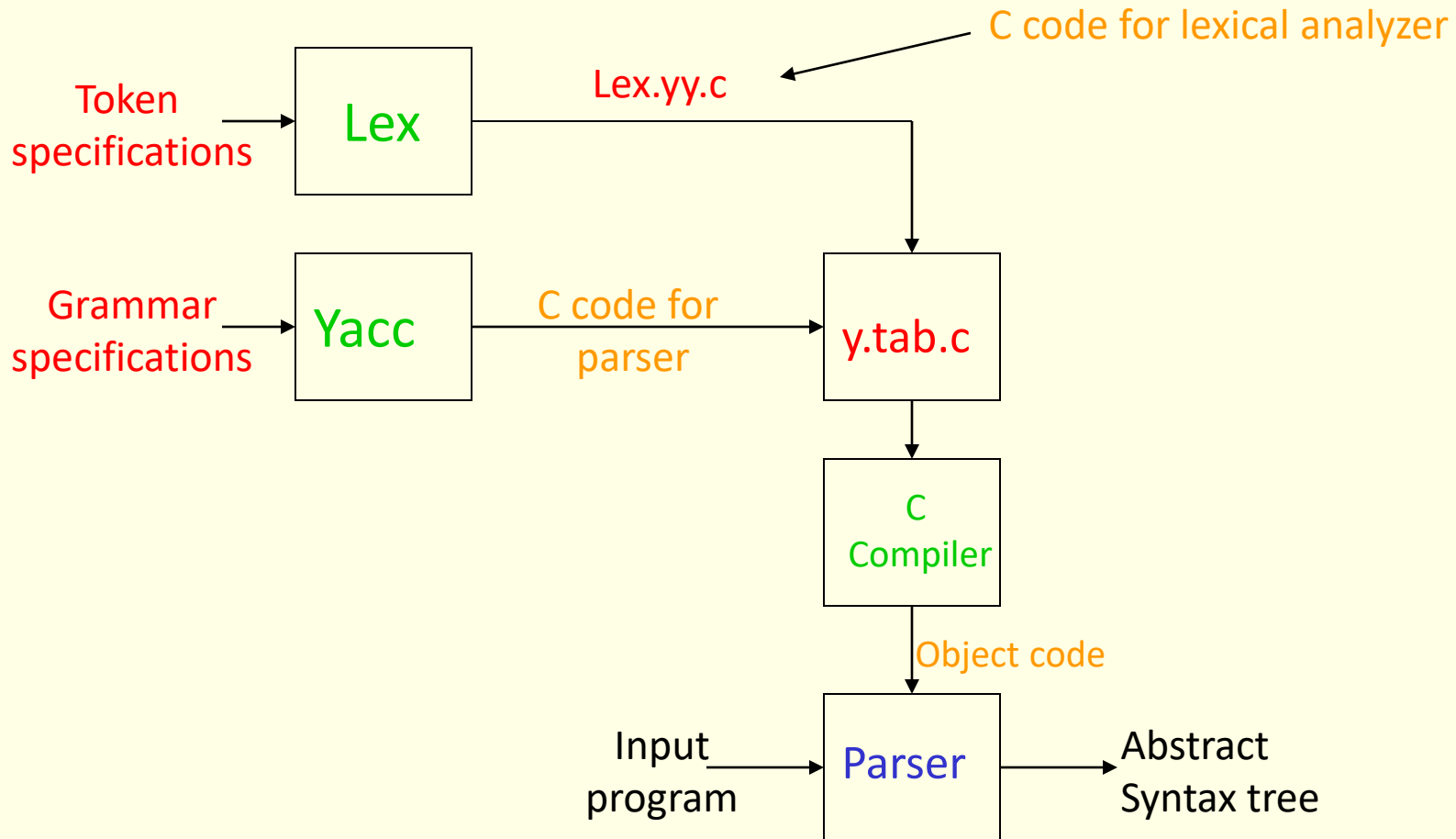
# Error Recovery

- An error is detected when an entry in the action table is found to be empty.
- Panic mode error recovery can be implemented as follows:
  - scan down the stack until a state  $S$  with a goto on a particular nonterminal  $A$  is found.
  - discard zero or more input symbols until a symbol  $a$  is found that can legitimately follow  $A$ .
  - stack the state  $\text{goto}[S,A]$  and resume parsing.
- **Choice of A:** Normally these are non terminals representing major program pieces such as an expression, statement or a block. For example if  $A$  is the nonterminal  $\text{stmt}$ ,  $a$  might be semicolon or end.

# Parser Generator

- Some common parser generators
  - YACC: **Y**et **A**nother **C**ompiler **C**ompiler
  - Bison: GNU Software
  - ANTLR: **A**nother **T**ool for **L**anguage **R**ecognition
- Yacc/Bison source program specification (accept LALR grammars)
  - declaration
    - %%
  - translation rules
    - %%
  - supporting C routines

# Yacc and Lex schema



Refer to YACC Manual

# Bottom up parsing ...

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax
- Automatic generation of parsers (Yacc, Bison etc.)
- Detects errors as soon as possible
- Allows better error recovery