## Top down Parsing

- Following grammar generates types of Pascal
type $\rightarrow$ simple
| 个id
| array [ simple] of type
simple $\rightarrow$ integer
| char
num dotdot num


## Example ...

- Construction of a parse tree is done by starting the root labeled by a start symbol
- repeat following two steps
- at a node labeled with non terminal $A$ select one of the productions of A and construct children nodes
(Which production?)
- find the next node at which subtree is Constructed
(Which node?)
- Parse
array [ num dotdot num ] of integer

- Cannot proceed as non terminal "simple" never generates a string beginning with token "array". Therefore, requires back-tracking.
- Back-tracking is not desirable, therefore, take help of a "look-ahead" token. The current token is treated as lookahead token. (restricts the class of grammars)



## Recursive descent parsing

First set:
Let there be a production

$$
A \rightarrow \alpha
$$

then First $(\alpha)$ is the set of tokens that appear as the first token in the strings generated from $\alpha$

For example :
First(simple) $=\{$ integer, char, num $\}$
First(num dotdot num) $=\{n u m\}$

## Define a procedure for each non terminal

```
procedure type;
    if lookahead in \{integer, char, num \}
    then simple
    else if lookahead \(=\uparrow\)
        then begin match ( \(\uparrow\) );
                match(id)
            end
        else if lookahead = array
            then begin match(array);
                match([);
                simple;
            match(]);
                        match(of);
                                type
            end
        else error;
```

procedure simple;
if lookahead = integer then match(integer) else if lookahead = char then match(char) else if lookahead = num then begin match(num); match(dotdot); match(num)
end else error;
procedure match(t:token);
if lookahead = t then lookahead = next token else error;

## Left recursion

- A top down parser with production $A \rightarrow A \alpha$ may loop forever
- From the grammar $A \rightarrow A \alpha \mid \beta$ left recursion may be eliminated by transforming the grammar to
$\mathrm{A} \rightarrow \beta \mathrm{R}$
$R \rightarrow \alpha R \mid \varepsilon$


## Parse tree corresponding

 to a left recursive grammar

Parse tree corresponding to the modified grammar

A

$\beta \quad \alpha$

Both the trees generate string $\beta \alpha^{*}$

## Example

- Consider grammar for arithmetic expressions

$$
\begin{aligned}
& \mathrm{E} \rightarrow \mathrm{E}+\mathrm{T} \mid \mathrm{T} \\
& \mathrm{~T} \rightarrow \mathrm{~T} * \mathrm{~F} \mid \mathrm{F} \\
& \mathrm{~F} \rightarrow(\mathrm{E}) \mid \mathrm{id}
\end{aligned}
$$

- After removal of left recursion the grammar becomes

$$
\begin{aligned}
& \mathrm{E} \rightarrow \mathrm{~T} \mathrm{E}^{\prime} \\
& \mathrm{E}^{\prime} \rightarrow+\mathrm{T} \mathrm{E}^{\prime} \mid \epsilon \\
& \mathrm{T} \rightarrow \mathrm{~F} \mathrm{~T}^{\prime} \\
& \mathrm{T}^{\prime} \rightarrow \mathrm{F}^{\prime} \mid \epsilon \\
& \mathrm{F} \rightarrow(\mathrm{E}) \mid \text { id }
\end{aligned}
$$

## Removal of left recursion

In general

$$
\begin{gathered}
A \rightarrow A \alpha_{1}\left|A \alpha_{2}\right| \ldots . . \mid A \alpha_{m} \\
\quad\left|\beta_{1}\right| \beta_{2}|\ldots \ldots| \beta_{n}
\end{gathered}
$$

transforms to

$$
\begin{aligned}
& A \rightarrow \beta_{1} A^{\prime}\left|\beta_{2} A^{\prime}\right| \ldots . . \mid \beta_{n} A^{\prime} \\
& A^{\prime} \rightarrow \alpha_{1} A^{\prime}\left|\alpha_{2} A^{\prime}\right| \ldots . .\left|\alpha_{m} A^{\prime}\right| \epsilon
\end{aligned}
$$

## Left recursion hidden due to many productions

- Left recursion may also be introduced by two or more grammar rules. For example:
$s \rightarrow A a \mid b$
$A \rightarrow A c|S d| \epsilon$
there is a left recursion because
$S \rightarrow A a \rightarrow S d a$
- In such cases, left recursion is removed systematically
- Starting from the first rule and replacing all the occurrences of the first non terminal symbol
- Removing left recursion from the modified grammar


## Removal of left recursion due to many productions ...

- After the first step (substitute $S$ by its rhs in the rules) the grammar becomes
$S \rightarrow A a \mid b$
$A \rightarrow A c|A a d| b d \mid \epsilon$
- After the second step (removal of left recursion) the grammar becomes
$S \rightarrow A a \mid b$
$\mathrm{A} \rightarrow \mathrm{bd} \mathrm{A}^{\prime} \mid \mathrm{A}^{\prime}$
$\mathrm{A}^{\prime} \rightarrow \mathrm{cA}\left|\mathrm{ad} \mathrm{A}^{\prime}\right| \Theta$


## Left factoring

- In top-down parsing when it is not clear which production to choose for expansion of a symbol
defer the decision till we have seen enough input.

In general if $A \rightarrow \alpha \beta_{1} \mid \alpha \beta_{2}$
defer decision by expanding $A$ to $\alpha A^{\prime}$
we can then expand $A^{\prime}$ to $\beta_{1}$ or $\beta_{2}$

- Therefore $A \rightarrow \alpha \beta_{1} \mid \alpha \beta_{2}$
transforms to
$A \rightarrow \alpha A^{\prime}$
$A^{\prime} \rightarrow \beta_{1} \mid \beta_{2}$


## Dangling else problem again

Dangling else problem can be handled by left factoring
stmt $\rightarrow$ if expr then stmt else stmt
| if expr then stmt
can be transformed to
stmt $\rightarrow$ if expr then stmt S'
$S^{\prime} \rightarrow$ else stmt \| $\in$

## Predictive parsers

- A non recursive top down parsing method
- Parser "predicts" which production to use
- It removes backtracking by fixing one production for every non-terminal and input token(s)
- Predictive parsers accept LL(k) languages
- First $L$ stands for left to right scan of input
- Second L stands for leftmost derivation
- $k$ stands for number of lookahead token
- In practice $\operatorname{LL}(1)$ is used


## Predictive parsing

- Predictive parser can be implemented by maintaining an external stack


Parse table is a two dimensional array $M[X, a]$ where " $X$ " is a non terminal and "a" is
a terminal of the grammar

## Example

- Consider the grammar

$$
\begin{aligned}
& \mathrm{E} \rightarrow \mathrm{~T} \mathrm{E}^{\prime} \\
& \mathrm{E}^{\prime} \rightarrow+\mathrm{T} \mathrm{E}^{\prime} \mid \epsilon \\
& \mathrm{T} \rightarrow \mathrm{FT}^{\prime} \\
& \mathrm{T}^{\prime} \rightarrow{ }^{*} \mathrm{FT}^{\prime} \mid \epsilon \\
& \mathrm{F} \rightarrow(\mathrm{E}) \mid \text { id }
\end{aligned}
$$

## Parse table for the grammar

|  | id | + | $*$ | $($ | $)$ | $\$$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $E \rightarrow T \mathrm{E}^{\prime}$ |  |  | $\mathrm{E} \rightarrow \mathrm{TE} \mathrm{E}^{\prime}$ |  |  |
| $\mathrm{E}^{\prime}$ |  | $\mathrm{E}^{\prime} \rightarrow+\mathrm{TE}$ |  |  | $\mathrm{E}^{\prime} \rightarrow \epsilon$ | $\mathrm{E}^{\prime} \rightarrow \epsilon$ |
| T | $\mathrm{T} \rightarrow \mathrm{FT}^{\prime}$ |  |  | $\mathrm{T} \rightarrow \mathrm{FT} \mathrm{T}^{\prime}$ |  |  |
| $\mathrm{T}^{\prime}$ |  | $\mathrm{T}^{\prime} \rightarrow \epsilon$ | $\mathrm{T}^{\prime} \rightarrow * \mathrm{~F}^{\prime}$ |  | $\mathrm{T}^{\prime} \rightarrow \epsilon$ | $\mathrm{T}^{\prime} \rightarrow \epsilon$ |
| F | $\mathrm{F} \rightarrow \mathrm{id}$ |  |  | $\mathrm{F} \rightarrow(\mathrm{E})$ |  |  |

Blank entries are error states. For example
E cannot derive a string starting with '+'

## Parsing algorithm

- The parser considers 'X' the symbol on top of stack, and 'a' the current input symbol
- These two symbols determine the action to be taken by the parser
- Assume that '\$' is a special token that is at the bottom of the stack and terminates the input string
if $X=a=\$$ then halt
if $\mathrm{X}=\mathrm{a} \neq \$$ then $\operatorname{pop}(\mathrm{x})$ and $\mathrm{ip++}$
if $X$ is a non terminal
then if $\mathrm{M}[\mathrm{X}, \mathrm{a}]=\{\mathrm{X} \rightarrow \mathrm{UVW}\}$ then begin pop(X); push(W,V,U) end else error


## Example

| Stack | input | action |
| :---: | :---: | :---: |
| \$E | id + id * id \$ | expand by $\mathrm{E} \rightarrow$ TE' |
| \$E'T | id + id * id \$ | expand by $T \rightarrow \mathrm{FT}^{\prime}$ |
| \$E'T'F | id + id * id \$ | expand by F $\rightarrow$ id |
| \$E'T'id | id + id * id \$ | pop id and ip++ |
| \$ $E^{\prime} \mathrm{T}^{\prime}$ | + id * id \$ | expand by $\mathrm{T}^{\prime} \rightarrow \epsilon$ |
| \$ $\mathrm{E}^{\prime}$ | + id * id \$ | expand by $\mathrm{E}^{\prime} \rightarrow+\mathrm{TE}^{\prime}$ |
| \$E'T+ | + id * id \$ | pop + and ip++ |
| \$E'T | id * id \$ | expand by $\mathrm{T} \rightarrow \mathrm{FT}^{\prime}$ |

## Example

| Stack | input | action |
| :---: | :---: | :---: |
| \$ $E^{\prime} T^{\prime} \mathrm{F}$ | id * id \$ | expand by F $\rightarrow$ id |
| \$E'T'id | id * id \$ | pop id and ip++ |
| \$E'T' | * id \$ | expand by $\mathrm{T}^{\prime} \rightarrow{ }^{*} \mathrm{FT}^{\prime}$ |
| \$E'T'F* | * id \$ | pop * and ip++ |
| \$ $E^{\prime} T^{\prime} \mathrm{F}$ | id \$ | expand by F $\rightarrow$ id |
| \$E'T'id | id \$ | pop id and ip++ |
| \$ $\mathrm{E}^{\prime}$ ' | \$ | expand by $\mathrm{T}^{\prime} \rightarrow \epsilon$ |
| \$E' | \$ | expand by $\mathrm{E}^{\prime} \rightarrow \epsilon$ |
| \$ | \$ | halt |

## Constructing parse table

- Table can be constructed if for every non terminal, every lookahead symbol can be handled by at most one production
- First( $\alpha$ ) for a string of terminals and non terminals $\alpha$ is
- Set of symbols that might begin the fully expanded (made of only tokens) version of $\alpha$
- Follow(X) for a non terminal X is
- set of symbols that might follow the derivation of $X$ in the input stream



## Compute first sets

- If $X$ is a terminal symbol then $\operatorname{First}(X)=\{X\}$
- If $X \rightarrow \epsilon$ is a production then $\epsilon$ is in $\operatorname{First}(X)$
- If $X$ is a non terminal
and $X \rightarrow Y_{1} Y_{2} \ldots Y_{k}$ is a production
then
if for some $i$, a is in $\operatorname{First}\left(Y_{i}\right)$ and $\epsilon$ is in all of First $\left(Y_{j}\right)$ (such that $\mathrm{j}<\mathrm{i}$ ) then a is in $\operatorname{First}(\mathrm{X})$
- If $\epsilon$ is in First $\left(Y_{1}\right)$... First $\left(Y_{k}\right)$ then $\epsilon$ is in $\operatorname{First}(X)$


## Example

- For the expression grammar

$$
\begin{aligned}
& \mathrm{E} \rightarrow \mathrm{~T} \mathrm{E}^{\prime} \\
& \mathrm{E}^{\prime} \rightarrow+\mathrm{T} \mathrm{E}^{\prime} \mid \epsilon \\
& \mathrm{T} \rightarrow \mathrm{~F} \mathrm{~T}^{\prime} \\
& \mathrm{T}^{\prime} \rightarrow \text { F T }^{\prime} \mid \epsilon \\
& \mathrm{F} \rightarrow(\mathrm{E}) \mid \text { id }
\end{aligned}
$$

$\operatorname{First}(E)=\operatorname{First}(T)=\operatorname{First}(F)=\{($, id $\}$
First( $\left.E^{\prime}\right)=\{+, \epsilon\}$
First( $\left.\mathrm{T}^{\prime}\right)=\left\{{ }^{*}, \Theta\right\}$

## Compute follow sets

1. Place $\$$ in follow(S)
2. If there is a production $A \rightarrow \alpha B \beta$
then everything in first( $\beta$ ) (except $\varepsilon$ ) is in follow(B)
3. If there is a production $\mathrm{A} \rightarrow \alpha \mathrm{B}$
then everything in follow(A) is in follow(B)
4. If there is a production $A \rightarrow \alpha B \beta$
and First $(\beta)$ contains $\varepsilon$ then everything in follow(A) is in follow(B)

Since follow sets are defined in terms of follow sets last two steps have to be repeated until follow sets converge

## Example

- For the expression grammar

$$
\begin{aligned}
& \mathrm{E} \rightarrow \mathrm{~T} \mathrm{E}^{\prime} \\
& \mathrm{E}^{\prime} \rightarrow+\mathrm{T} \mathrm{E}^{\prime} \mid \epsilon \\
& \mathrm{T} \rightarrow \mathrm{~F} \mathrm{~T}^{\prime} \\
& \mathrm{T}^{\prime} \rightarrow \text { F T }^{\prime} \mid \epsilon \\
& \mathrm{F} \rightarrow(\mathrm{E}) \mid \text { id }
\end{aligned}
$$

follow $(E)=$ follow $\left.\left(E^{\prime}\right)=\{\$),\right\}$
follow $(T)=$ follow $\left.\left(T^{\prime}\right)=\{\$),+,\right\}$
follow(F) = \{\$, ), +, *\}

## Construction of parse table

- for each production $\mathrm{A} \rightarrow \alpha$ do
- for each terminal ' $a$ ' in first( $\alpha$ )

$$
\mathrm{M}[\mathrm{~A}, \mathrm{a}]=\mathrm{A} \rightarrow \alpha
$$

- If $\epsilon$ is in $\operatorname{First}(\alpha)$
$\mathrm{M}[\mathrm{A}, \mathrm{b}]=\mathrm{A} \rightarrow \alpha$
for each terminal $b$ in follow( $A$ )
- If $\varepsilon$ is in First( $\alpha$ ) and $\$$ is in follow(A)
$\mathrm{M}[\mathrm{A}, \$ \mathrm{~S}]=\mathrm{A} \rightarrow \alpha$
- A grammar whose parse table has no multiple entries is called $\operatorname{LL}(1)$


## Practice Assignment

- Construct LL(1) parse table for the expression grammar bexpr $\rightarrow$ bexpr or bterm | bterm bterm $\rightarrow$ bterm and bfactor | bfactor bfactor $\rightarrow$ not bfactor \| (bexpr ) | true \| false
- Steps to be followed
- Remove left recursion
- Compute first sets
- Compute follow sets
- Construct the parse table


## Error handling

- Stop at the first error and print a message
- Compiler writer friendly
- But not user friendly
- Every reasonable compiler must recover from errors and identify as many errors as possible
- However, multiple error messages due to a single fault must be avoided
- Error recovery methods
- Panic mode
- Phrase level recovery
- Error productions
- Global correction


## Panic mode

- Simplest and the most popular method
- Most tools provide for specifying panic mode recovery in the grammar
- When an error is detected
- Discard tokens one at a time until a set of tokens is found whose role is clear
- Skip to the next token that can be placed reliably in the parse tree


## Panic mode

- Consider following code begin

$$
\begin{aligned}
& a=b+c ; \\
& x=p r ; \\
& h=x<0 ;
\end{aligned}
$$

end;

- The second expression has syntax error
- Panic mode recovery for begin-end block skip ahead to next ';' and try to parse the next expression
- It discards one expression and tries to continue parsing
- May fail if no further ';' is found


## Phrase level recovery

- Make local correction to the input
- Works only in limited situations
- A common programming error which is easily detected
- For example insert a ";" after closing "\}" of a class definition
- Does not work very well!


## Error productions

- Add erroneous constructs as productions in the grammar
- Works only for most common mistakes which can be easily identified
- Essentially makes common errors as part of the grammar
- Complicates the grammar and does not work very well


## Global corrections

- Considering the program as a whole find a correct "nearby" program
- Nearness may be measured using certain metric
- PL/C compiler implemented this scheme: anything could be compiled!
- It is complicated and not a very good idea!


## Error Recovery in LL(1) parser

- Error occurs when a parse table entry $\mathrm{M}[\mathrm{A}, \mathrm{a}]$ is empty
- Skip symbols in the input until a token in a selected set (synch) appears
- Place symbols in follow(A) in synch set. Skip tokens until an element in follow(A) is seen.
Pop(A) and continue parsing
- Add symbol in first(A) in synch set. Then it may be possible to resume parsing according to A if a symbol in first(A) appears in input.


## Practice Assignment

- Reading assignment: Read about error recovery in LL(1) parsers
- Assignment to be submitted:
- introduce synch symbols (using both follow and first sets) in the parse table created for the boolean expression grammar in the previous assignment
- Parse "not (true and or false)" and show how error recovery works

